Ватутин Э.И.[1]**, Белышев А.Д.**[2]**, Никитина Н.Н.**[3]**, Манзюк М.О.**[2]

[1] -
[2] - BOINC.ru
[3]

ИСПОЛЬЗОВАНИЕ X-ОБРАЗНЫХ ДИАГОНАЛЬНЫХ ЗАПОЛНЕНИЙ И ESODLS CMS СХЕМ ДЛЯ ПЕРЕЧИСЛЕНИЯ ГЛАВНЫХ КЛАССОВ ДИАГОНАЛЬНЫХ ЛАТИНСКИХ КВАДРАТОВ

Ключевые слова: , , , , , OEIS, BOINC.

Аннотация. X- ( ), , .

ESODLS CMS , 3–4 .

Gerasim@Home BOINC 9, 3 292 326 155 394.

Введение

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[1].
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BOINC [4]:
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Основные понятия и определения

N $A=\left[a_{ij}\right]$

$N\times N$, $i=\overline{0,N-1}$, $j=\overline{0,N-1}$,

$U=\{0,1,...,N-1\}$,

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$N!$ N,

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$N!$.

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(. Cells Mapping Schema, . CMS)

[5] p . ,

$\left(p[i]=i\right)$, .

M- CMS

, *ESODLS CMS* (

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ESODLS CMS, [14] - [7]

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Понятие X-образных диагональных заполнений

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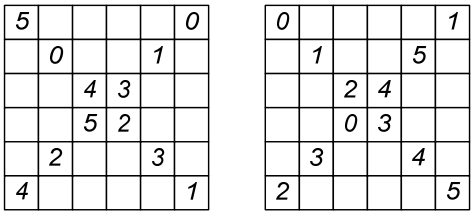


. 1. X- 6 ( )
( )

X-
N. $N \le 13$
, $N = 14$
$N = 15$ 1,5 Core i7 4770 1 ,
:

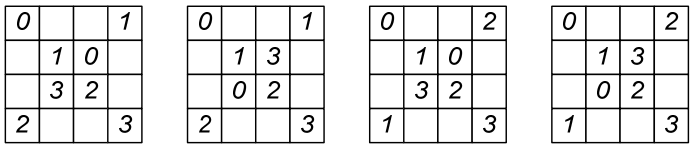
1, 0, 0, 4, 4, 80, 80, 4752, 4752, 440192, 440192, 59245120, 59245120, 10930514688, 10930514688.

(
), -
(. OEIS) [8] A337302[1].
$N!$, , X-
:

1, 0, 0, 96, 480, 57600, 403200, 191600640, 1724405760, 1597368729600, 17571056025600, 28378507272192000, 368920594538496000, 952903592436341145600, 1429355388654511718400 0.

OEIS,
A337303. ,
$A337302(N) \cdot N! = A337303(N)$.

4 . 2.



. 2. 4
$(A337302(4) = 4)$.

, A337302
, :
$A337302(2t) = A337302(2t+1)$ $t > 0$. ,
$t \ge 0$ ($A337302(0) = A337302(1) = 1$).
$N = 2t$.
$0, 1, ..., 2t-1$ ($2t$).
$U_1 = \{0, 1, ..., 2t-1\}$ $|U_1| = 2t$
, $N = 2t$

[1] https://oeis.org/A337302

U_1:

, – .

$N = 2t+1$.

$0, 1, ..., 2t$ ($2t+1$),

( ) ,

.

$N-1=2t$, .

,

, $U_2 = \{0, 1, ..., 2t\} \setminus \{t\} = \{0, 1, ..., t-1, t+1, ..., 2t\}$

$|U_2| = 2t$, U_1 U_2 .

( ).

, $2t$ $2t+1$

: $2t$ $2t$

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U_1 U_2, .

, , .

1, 0, 4, 80, 4752, 440192, 59245120, 10930514688,

OEIS A000316,

[9–11],

A337302 **o**

,

$A337302(N) = A000316\left(\left\lfloor \frac{N}{2} \right\rfloor\right)$ (Andrew

Howroyd Alois P. Heinz ,

OEIS).

Классы эквивалентности X-образных диагональных заполнений

X-

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X- , , X-

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X-

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M- (, , – ESODLS CMS)

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X-

. N

1, 0, 0, 2, 2, 3, 3, 20, 20, 67, 67, 596, 596,

OEIS,

A309283.

, $A309283(2t) = A309283(2t+1)$ $t \geq 0$
, M- (ESODLS CMS) ,

$2t$ $2t+1$.
OEIS A338084. $N \leq 10$
, $N = 11$
2,5 Core i7 4770 1 ,
$N = 12$ $N = 13$ – 2
Gerasim@Home[2]
7 TFLOP/s
(Harry White
).
X-
, ,
. M-

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X- M-
(. . 3).

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0	1	2	3	4	5	6	7
1	2	0	4	5	7	3	6
6	7	3	0	1	4	5	2
4	6	7	5	2	1	0	3
2	4	5	6	7	3	1	0
3	5	1	2	0	6	7	4
7	0	6	1	3	2	4	5
5	3	4	7	6	0	2	1

)

0	7	3	6	5	4	1	2
3	1	6	2	7	0	5	4
1	4	2	0	6	3	7	5
5	6	7	3	1	2	4	0
2	5	0	7	4	6	3	1
7	2	4	1	3	5	0	6
4	0	1	5	2	7	6	3
6	3	5	4	0	1	2	7

)

0	1	2	6	4	5	3	7
2	3	6	7	1	0	4	5
3	5	7	0	6	2	1	4
4	6	1	2	3	7	5	0
7	4	0	1	5	6	2	3
1	7	5	3	2	4	0	6
5	0	3	4	7	1	6	2
6	2	4	5	0	3	7	1

)

0	2	3	5	6	4	7	1
5	1	4	6	7	0	2	3
4	6	2	7	0	3	1	5
2	7	1	3	5	6	0	4
7	3	6	0	4	1	5	2
3	0	7	1	2	5	4	6
1	4	5	2	3	7	6	0
6	5	0	4	1	2	3	7

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(15)

. 3. :

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[2] https://gerasim.boinc.ru

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15-

X- $K_1 = A274171(N) = \sum_{i=1}^{m} |C_i| f_i$,

$m = A309283(N)$ – X-

(N), $|C_i|$ – i- (. .

X- , M-

X-

), f_i –

X- (

, .), $K_2 = A274806(N) = K_1 \cdot N!$

. [12]

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4 5 . 4.

0			1
	1	0	
	3	2	
2			

1	[1,0,2,4,3]	2	2	$2\times 2=4$
2	[1,4,2,0,3]	2	2	$2\times 2=4$
:				8

3. $N=6$

i		$\lvert C_i\rvert$	f_i	$\lvert C_i\rvert\times f_i$
1	[1,0,4,5,2,3]	16	8	128
2	[1,0,4,5,3,2]	48	0	0
3	[1,2,0,5,3,4]	16	0	0
:				128

4. $N=7$

i		$\lvert C_i\rvert$	f_i	$\lvert C_i\rvert\times f_i$
1	[1,0,5,3,6,2,4]	16	1 800	28 800
2	[1,0,5,3,6,4,2]	48	2 008	96 384
3	[1,2,0,3,6,4,5]	16	2 876	46 016
:				171 200

5. $N=8$

i		$\lvert C_i\rvert$	f_i	$\lvert C_i\rvert\times f_i$
1	[1,0,3,2,5,4,7,6]	12	855 680	10 268 160
2	[1,0,3,2,5,6,7,4]	192	1 087 936	208 883 712
3	[1,0,3,2,6,7,4,5]	96	2 079 952	199 675 392
4	[1,0,3,2,6,7,5,4]	192	1 701 792	326 744 064
5	[1,0,3,5,2,4,7,6]	24	607 872	14 588 928
6	[1,0,3,5,2,6,7,4]	384	1 075 784	413 101 056
7	[1,0,3,5,6,7,2,4]	768	1 673 128	1 284 962 304
8	[1,0,3,5,6,7,4,2]	384	1 621 760	622 755 840
9	[1,0,3,6,5,7,2,4]	192	1 714 248	329 135 616
10	[1,0,3,6,5,7,4,2]	192	1 537 024	295 108 608
11	[1,0,3,6,7,4,2,5]	384	1 675 696	643 467 264
12	[1,0,3,6,7,4,5,2]	384	1 739 980	668 152 320
13	[1,2,0,5,6,7,3,4]	384	1 678 124	644 399 616
14	[1,2,3,0,7,4,5,6]	48	1 607 168	77 144 064
15	[1,2,3,5,0,7,4,6]	384	1 656 184	635 974 656
16	[1,2,3,5,6,7,0,4]	384	1 649 308	633 334 272
17	[1,2,3,5,7,4,0,6]	192	1 072 016	205 827 072
18	[1,2,3,7,0,4,5,6]	48	1 472 416	70 675 968
19	[1,2,6,5,7,4,0,3]	96	1 641 968	157 628 928
20	[1,7,3,5,2,4,0,6]	12	480 000	5 760 000
:				7 447 587 840

6. $N=9$

i		$\lvert C_i \rvert$	f_i	$\lvert C_i \rvert \times f_i$
1	[1,0,3,2,4,6,5,8,7]	12	709 086 781 440	8 509 041 377 280
2	[1,0,3,2,4,6,7,8,5]	192	974 677 273 856	187 138 036 580 352
3	[1,0,3,2,4,7,8,5,6]	96	1 055 969 508 864	101 373 072 850 944
4	[1,0,3,2,4,7,8,6,5]	192	1 110 308 250 368	213 179 184 070 656
5	[1,0,3,6,4,2,5,8,7]	24	685 740 255 232	16 457 766 125 568
6	[1,0,3,6,4,2,7,8,5]	384	955 718 665 632	366 995 967 602 688
7	[1,0,3,6,4,7,8,2,5]	768	1 098 599 343 136	843 724 295 528 448
8	[1,0,3,6,4,7,8,5,2]	384	1 031 276 411 456	396 010 141 999 104
9	[1,0,3,7,4,6,8,2,5]	192	1 110 075 761 408	213 134 546 190 336
10	[1,0,3,7,4,6,8,5,2]	192	1 029 535 024 256	197 670 724 657 152
11	[1,0,3,7,4,8,5,2,6]	384	1 116 973 550 464	428 917 843 378 176
12	[1,0,3,7,4,8,5,6,2]	384	1 117 216 705 792	429 011 215 024 128
13	[1,2,0,6,4,7,8,3,5]	384	1 105 080 908 576	424 351 068 893 184
14	[1,2,3,0,4,8,5,6,7]	48	1 081 071 822 848	51 891 447 496 704
15	[1,2,3,6,4,0,8,5,7]	384	1 105 414 783 232	424 479 276 761 088
16	[1,2,3,6,4,7,8,0,5]	384	1 086 754 151 040	417 313 593 999 360
17	[1,2,3,6,4,8,5,0,7]	192	937 027 424 256	179 909 265 457 152
18	[1,2,3,8,4,0,5,6,7]	48	1 081 947 743 232	51 933 491 675 136
19	[1,2,7,6,4,8,5,0,3]	96	1 010 590 517 376	97 016 689 668 096
20	[1,8,3,6,4,2,5,0,7]	12	664 832 014 336	7 977 984 172 032
:				5 056 994 653 507 584

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$$N\colon \sum_{i=1}^{m} \lvert C_i \rvert = A337302(N).$$

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Gerasim@Home , 2 .
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Генератор СКФ ДЛК

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$C_{\max}$,).
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ESODLS

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X- X_0

X- $X_0 \xrightarrow{ESODLS\ CMS_i} X'$

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1. X- X' : $X_0 = X'$;
2. X- ,
: $X_0 \neq X'$.

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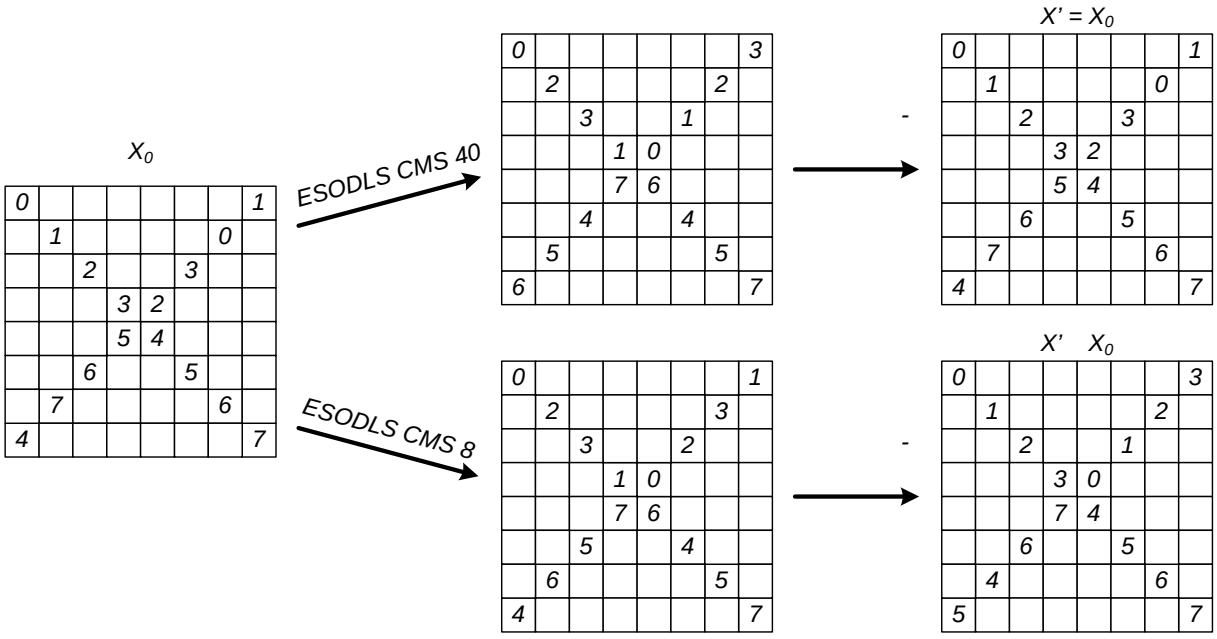

. 5. ESODLS , X-

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X-

ESODLS ,

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$|C_i|$ *i*, , $\frac{C_{\max}}{|C_i|}$ (

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(ESODLS)

$C_{\max}$ $\frac{C_{\max}}{|C_i|}-1$, . . $\eta=\frac{C_{\max}}{\frac{C_{\max}}{|C_i|}-1}\approx|C_i|$ (–1 ,

ESODLS CMS , . . X-

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$|C_i|=C_{\max}$

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. 7–11.

7. $N=4$

i	$\lvert C_i\rvert$	f_i	n_i
1	2	1	1
2	2	0	0
( )			1

8. $N=5$

i	$\lvert C_i\rvert$	f_i	n_i
1	2	2	1
2	2	2	1
( )			2

9. $N=6$

i	$\lvert C_i\rvert$	f_i	n_i
1	16	8	2
2	48	0	0
3	16	0	0
( )			2

10. $N=7$

i	$\lvert C_i\rvert$	f_i	n_i
1	16	1 800	181
2	48	2 008	530
3	16	2 876	261
( )			972

11. $N=8$

i	$\lvert C_i\rvert$	f_i	n_i	. / ,
1	12	855 680	8 148	30

2	192	1 087 936	137 801	843
3	96	2 079 952	135 595	1 926
4	192	1 701 792	213 166	2 530
5	24	607 872	10 092	50
6	384	1 075 784	270 633	5 146
7	768	1 673 128	837 748	3 723
8	384	1 621 760	405 668	35 455
9	192	1 714 248	214 433	3 369
10	192	1 537 024	193 044	847
11	384	1 675 696	419 434	3 889
12	384	1 739 980	437 267	1 669
13	384	1 678 124	421 525	2 675
14	48	1 607 168	51 530	207
15	384	1 656 184	414 374	11 826
16	384	1 649 308	412 695	2 318
17	192	1 072 016	135 729	908
18	48	1 472 416	46 301	1 858
19	96	1 641 968	103 873	543
20	12	480 000	4 040	26
(		)	4 873 096	

(. A287764 OEIS) [3].

$$\frac{f_i}{n_i} \approx \frac{C_{\max}}{|C_i|},$$

.

, ,

: $n_i \approx \frac{1}{\underbrace{C_{\max}}_{const}} \times |C_i| f_i$.

Подсчет главных классов ДЛК порядка 9

9.

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(. Work Unit, . WU)

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$d = 10$. 10 (

WU')

WU' (

WU'),

d 633 300.

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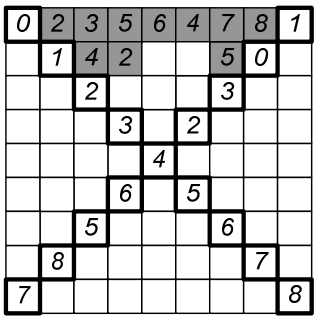

. 6. , :

X- , $-d$

j-

: a_j

ESODLS

b_j. . 6 $a_j = 1\,536\,075$

$b_j = 12\,783\,424$.

7 TFLOP/s

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a_j b_j

. $\sum_j a_j$

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) – $\sum_j b_j$.

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12. $N = 9$

i	$\lvert C_i \rvert$		$f_i = \sum_j b_j$	$n_i = \sum_j a_j$
1	12	29 148	709 086 781 440	5 540 161 690
2	192	30 210	974 677 273 856	121 835 130 652
3	96	30 946	1 055 969 508 864	65 998 993 664
4	192	30 902	1 110 308 250 368	138 788 845 312
5	24	29 346	685 740 255 232	10 714 968 132
6	384	30 752	955 718 665 632	238 930 442 470
7	768	31 669	1 098 599 343 136	549 299 969 552
8	384	31 178	1 031 276 411 456	257 819 693 456

9	192	31 336	1 110 075 761 408	138 760 018 564
10	192	30 892	1 029 535 024 256	128 692 729 396
11	384	31 175	1 116 973 550 464	279 243 953 632
12	384	31 222	1 117 216 705 792	279 304 740 800
13	384	33 109	1 105 080 908 576	276 271 823 858
14	48	34 955	1 081 071 822 848	33 784 169 152
15	384	33 672	1 105 414 783 232	276 354 111 228
16	384	33 966	1 086 754 151 040	271 688 772 740
17	192	32 794	937 027 424 256	117 129 393 532
18	48	34 323	1 081 947 743 232	33 811 009 926
19	96	32 348	1 010 590 517 376	63 163 016 100
20	12	29 357	664 832 014 336	5 194 211 538
			( )	3 292 326 155 394

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citerra SerVal (Russia Team) - *BOINC.ru*
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