

Mtg 6: Tue, 12 Jan 10

16-1

HW: $f(x) = \sin x$, $x \in [0, \pi]$

Constr. Taylor Series of $f(\cdot)$ around $x_0 = \frac{\pi}{4}$ for $n = 0, 1, \dots, 10$.

Plot these series (for each n).

$R(x)$: Find (estimate the max) of $R(x)$ at $x = \frac{\pi}{2}$.

$$R(x) = \frac{1}{n!} \int_{x_0}^x (x-t)^n f^{(n+1)}(t) dt$$

$$= \frac{1}{n!} f^{(n+1)}(\xi) \int_{x_0}^x (x-t)^n dt$$

$$= \frac{(x-x_0)^{n+1}}{(n+1)!} f^{(n+1)}(\xi)$$

$$|R(x)| \leq \frac{(x-x_0)^{n+1}}{(n+1)!} \max_{\substack{\xi \in [x_0, x] \\ t \in [x_0, x]}} |f^{(n+1)}(t)|$$

≤ 1

Note: Motivation for 3f of $\underline{L5-2}$
Taylor series expansion (similar
tech will be used):

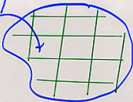
- * higher order error analysis of
trap. rule (not in A.V.)
- * Richardson extrap.
- * Clenshaw - Curtis quadrature
- * Chebyshev poly (orthogonal)
- * Recent devel. using Chebyshev
poly. to solve LD-ODE-VC
(Lin. 2nd order Ordin. Diff.
Eg. Varying Coeff.)
- * comb. of symbolic + numeric

Note: Quadrature = num. int. [≡]
 ↑ Quadrilateral
Greeks: Meas. area

Quadrilateral

6-3

Cubature
cube



$$\text{Area} = \sum \text{Quad.} \quad (1)$$

$$\text{Vol} = \sum \text{cubes} \quad (2)$$

Num. Int. using Taylor series
Cont'd p. 2-2 (1)

$$I_n = \int_0^1 f_n(x) dx = \int_0^1 \sum_{j=1}^n \frac{x^{j-1}}{j!} dx$$

$$= \sum_{j=1}^n \frac{1}{j!} \frac{1}{j} \quad (3)$$

$$(4) \quad f(x) - f_n(x) = R_n(x) = \frac{(x-0)^n}{(n+1)!} \uparrow$$

$\xi \in [0, x]$ exp [5]

ξ is a func. of $x \Rightarrow \xi_x = \xi(x)$

(6-4)

$$E_n := I - I_n$$

$$= \int_0^1 [f(x) - f_n(x)] dx$$

$$= \int_0^1 \underbrace{\frac{x^n}{(n+1)!}}_{w(x)} \underbrace{\exp[\xi(x)]}_{g(x)} dx$$

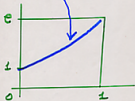
IMVT

$$\downarrow \Rightarrow \underbrace{g(\alpha)}_{\text{"}} \underbrace{\int_0^1 w(x) dx}_{1}, \quad \alpha \in [0, 1]$$

$$\exp[\xi(\alpha)]$$

$$\xi \in [0, 1]$$

$$\frac{1}{(n+1)!(n+1)}$$



$$\min_{\alpha \in [0, 1]} g(\alpha) = 1$$

$$\alpha \in [0, 1]$$

$$\max_{\alpha \in [0, 1]} g(\alpha) = e$$

$$\alpha \in [0, 1]$$

$$\frac{1}{(n+1)!(n+1)} \leq E_n \leq \frac{e}{(n+1)!(n+1)}$$

$$I_6 = 1.3178... \quad \text{A. p.250} \quad \underline{\underline{6.5}}$$

$$2.83 \times 10^{-5} \leq E_6 = I - I_6 \leq 7.70 \times 10^{-5}$$

HW: $I = \int_0^1 \frac{e^x - 1}{x} dx$

Use 3 methods to find I_n :

- 1) Taylor series exp. f_n
- 2) Comp. Trap. rule
- 3) Comp. Simpson rule

$n = 2, 4, 8, \dots$ until error of order 10^{-6} .