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Brief paper

Design of noise and period-time robust high-order repetitive control, with application to optical storage☆

Maarten Steinbuch[a,*], Siep Weiland[b], Tarunraj Singh[c]

[a]Control Systems Technology Group, Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB, Eindhoven, Netherlands
[b]Control Systems Group, Department of Electrical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[c]Department of Mechanical and Aerospace Engineering, SUNY at Buffalo, Buffalo, NY 14260, USA

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Abstract

Repetitive control is useful if periodic disturbances or setpoints act on a control system. Perfect (asymptotic) disturbance rejection is achieved if the period time is exactly known. The improved disturbance rejection at the periodic frequency and its harmonics is achieved at the expense of a degraded system sensitivity at intermediate frequencies. A convex optimization problem is defined for the design of high-order repetitive controllers, where a trade-off can be made between robustness for changes in the period time and for reduction of the error spectrum in-between the harmonic frequencies. The high-order repetitive control algorithms are successfully applied in experiments with the tracking control of a CD-player system.

Keywords: Repetitive control; Compact disc player; Periodic disturbances; Internal model principle

1. Introduction

Control systems subject to periodic disturbances may well benefit from the use of *repetitive control* (Hara, Yamamoto, Omata, & Nakano, 1988; Tomizuka, Tsao, & Chew, 1988). Repetitive controllers employ the internal model principle (Francis & Wonham, 1975; Rogers & Owens, 1992; de Roover, Bosgra, & Steinbuch, 2000) and consist of a periodic signal generator, enabling perfect (asymptotic) rejection of periodic disturbances. Applications are for instance known in magnetic and optical storage devices (Bodson, Sacks, & Khosla, 1994; Chen, Ding, Xiu, Ooi, & Tan, 2003; Chew & Tomizuka, 1990; Choi, Oh, & Choi, 1999; Dötch, Smakman, Van den Hof, & Steinbuch, 1995; Guo, 1997; Li & Tsao, 1999; Moon, Lee, & Chung, 1998; Messner & Bodson, 1994) and other motion systems and mechanics (Choi, Lim, & Choi, 2002; Fung, Huang, Chien, & Wang, 2000; Gotou, Ueta, Nakamura, & Matsuo, 1991; Hillerström, 1996; Kim & Tsao, 2000; Manayathara, Tsao, Bentsman, & Ross, 1996; Yau & Tsai, 1999).

One of the drawbacks of repetitive control is the requirement of exact knowledge of the period time of the external signals (Luo & Mahawan, 1998; Steinbuch, 2002; Tsao & Nemani, 1992). This means that in practical applications, either the period time is required to be constant, or an accurate measurement of the periodicity is necessary. Another drawback is due to the Bode sensitivity integral: the perfect reduction at the harmonic frequencies is counteracted by amplification of noise at intermediate frequencies.

Various solutions have been presented in the literature addressing this problem. The classical trade-off between robustness, performance and noise sensitivity is discussed in Guo (1997), Kim, Li, and Tsao (2004), Köroğlu and Morgü (1999), Lee and Smith (1998), Li and Tsao (1999), Yamada, Riadh, and Funahashi, (1999) using H_∞ and LQ-based repetitive controllers. Time-varying and adaptive repetitive control is introduced in Cao and Ledwich (2001), Köroğlu and Morgül (2001)

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* Corresponding author.
E-mail addresses: m.steinbuch@tue.nl (M. Steinbuch), s.weiland@tue.nl (S. Weiland), tsingh@eng.buffalo.edu (T. Singh).

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and Xu and Yao (2001). Recently, approaches have been reported in the literature making use of *high-order periodic signal generators* (Chang, Such, & Kim, 1995; Inoue, 1990;Schootstra & Steinbuch, 1998; Steinbuch & Schootstra, 1998; Steinbuch, 2002). In Steinbuch (2002) an approach is presented to design a high-order repetitive controller (HORC) such that robustness for period changes is obtained. In Steinbuch (2002) a constraint optimization problem is solved such that a desensitizing effect is obtained for non-repetitive signals. Approaches to make iterative learning schemes robustly stable with respect to iteration dependent disturbances and uncertainties have been reported in Chen and Moore (2002) and Moore and Chen (2002). For systems with multiple periodic signals a solution is presented in Yamada et al. (2000) and Chang, Suh, and Oh (1998).

In this paper we will generalize the results of Chang et al. (1995) and show that it can be cast into a convex optimization problem. We will also show that, using appropriate weighting functions, the same problem formulation can yield both the results of Steinbuch (2002) and the ones in Chang et al. (1995). The paper is an extended version of Steinbuch, van den Eerenbeemt, Weiland, and Singh (2004).

In Section 2 we will introduce the structure of high-order repetitive controllers, and show how stability can be guaranteed. In Section 3 the new optimization problem will be formulated. In Section 4 an application to a CD-player mechanism will be shown. Main results will be summarized in the form of conclusions in Section 5.

2. High-order repetitive control

Consider the general repetitive control system shown in Fig. 1. The *repetitive controller* is shown in the figure as the device $M(z)$, which includes a memory loop or delay line (Steinbuch, 2002). In *high-order repetitive control*, the total delay of the memory loop is extended to an integer multiple p of N samples by connecting multiple delays in series in a structure as shown in Fig. 2. Note that in this block diagram a *noncausal robustness filter* Q and a *learning filter* L are incorporated. Their respective delays are compensated by the structure, making the filtering noncausal, which is realized as follows. The filter Q is designed as a linear phase filter with q samples delay, which are then compensated by removing q samples from the memory buffer (block $z^{(N-g-l)}$). The total effect of the filtering with Q is then zero-phase. The filter L is designed using, for instance, the zero phase error tracking controller (ZPETC) algorithm as proposed in Tomizuka (1987), with a phase delay of l samples, and l delays are shifted from the forward path in the memory buffer to the feedback part, such that the total amount of delays is still correct, but the ones from L are compensated in the path.

Notice that because of the use of multiple delay lines, the control signal can be computed as a weighted sum of the signals of one, two, and more periods ago.

The relation between the input e and the output z of the HORC of Fig. 2 is described by the transfer function

$$M(z) = \frac{L(z)Q(z)W(z)z^{-(N-l-q)}}{1-Q(z)W(z)z^{-(N-q)}}, \tag{1}$$

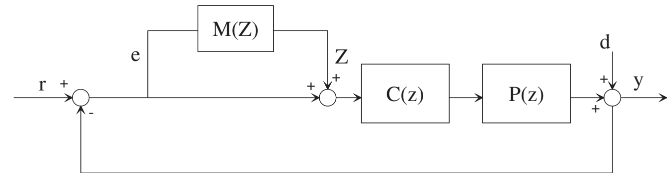

Fig. 1. Block scheme of the general repetitive controller.

where W is the *gain adjusting* or *high-order repetitive function*, given by

$$W(z) = \sum_{i=1}^{p} w_i z^{-(i-1)N} \tag{2}$$

with $\sum_{i=1}^{p} w_i = 1$, which ensures infinite gain at the harmonic frequencies.

2.1. Stability

Consider the control system of Fig. 1, with $M(z)$ the HORC of Fig. 2. The corresponding sensitivity function is given by $S{=}1/(1{+}PC(1{+}M))$. To analyze the stability of the controlled system, (1) is used to obtain

$$\begin{aligned} S &= \frac{1}{1+PC\left(1+(LQWz^{-(N-q-l)})/(1-QWz^{-(N-q)})\right)} \\ &= \frac{1}{1+PC}\cdot M_S, \end{aligned} \tag{3}$$

where M_S is the *modifying sensitivity function* (or *relative sensitivity error transfer function* Chang et al. (1995)). M_S is given by

$$M_S(z) := \frac{1-QWz^{-(N-q)}}{1-QWz^{-(N-q)}(1-TLz^{+l})}, \tag{4}$$

where T is the complementary sensitivity $T = PC/(1+PC)$. Hence M_S modifies the standard sensitivity function as a result of the repetitive control action.

Using the expression for M_S, a sufficient (small gain) criterion for stability now becomes

$$|W(z)Q(z)z^{-(N-q)}(1-T(z)L(z)z^{+l})| < 1 \tag{5}$$

for all z with $|z| = 1$.

2.2. Performance

In order to analyze the performance of the repetitive controller for various design choices (i.e. order p and weighting parameters w_i), we will focus within the passband of the robustness filter Q, and hence assume that $Q = 1$ and $q = 0$. Furthermore, we assume that $LT \approx k_r$, with (k_r the *learning gain*) and also $l = 0$. That is, for low frequencies, where the inverse of T is normally exactly known, k_r chosen close to or equal to one.

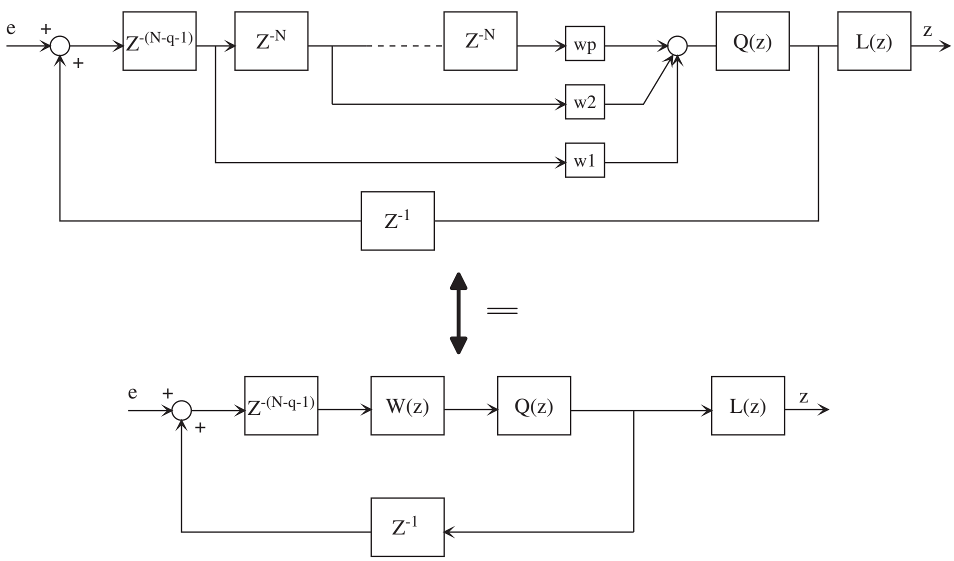

Fig. 2. Block scheme of the high-order repetitive controller $M(z)$.

Under these assumptions, Eq. (4) simplifies to

$$M_S(z) = \frac{1 - W(z)z^{-N}}{1 - W(z)z^{-N}(1 - k_r)}. \tag{6}$$

Furthermore, to make the analysis independent of the period time of the disturbance, the normalized frequency $\theta = \omega N T_s$ with T_s the sampling time is introduced. With the substitution $z = e^{j\omega T_s}$ or $z = e^{sT_s}$, the modifying sensitivity function in Eq. (6) becomes a function of the normalized frequency θ and is (with some abuse of notation) given by

$$M_S(\theta) = \frac{1 - W(\theta)e^{-j\theta}}{1 - W(\theta)e^{-j\theta}(1 - k_r)}, \tag{7}$$

where

$$W(\theta) = \sum_{i=1}^{p} w_i e^{-(i-1)j\theta}.$$

is the (normalized) high-order repetitive function. Remark that for the normalized frequency θ we have that $2\pi(k-1) \leqslant \theta \leqslant 2\pi k$. Since the expressing is a periodic function, in the remainder of the paper we take $k = 1$, and hence choose the range as $0 \leqslant \theta \leqslant 2\pi$.

3. Optimization design problem

In Inoue (1990), Inoue proposes the weighting factors in (2) as $w_i = 1/p$ and shows that this choice minimizes the averaged square of $|M_S(z)|$ over all frequencies. In Chang et al. (1995), the authors propose an 'evolution strategy' to minimize the H_∞ norm of M_S over all weighting factors w_i, assuming the gain k_r to be fixed. Here we will extend the approach of Chang et al. (1995) in two ways: first we will omit the unnecessary constraint (Inoue, 1990) that each weighting factor $w_i \leqslant 1$ (but we retain the condition on the sum!), and secondly we will show that the design optimization problem can be rephrased and solved by a linear programming algorithm. The weighting factors of the high-order repetitive function W defined in (2) are determined in such a way that the infinity norm of the modifying sensitivity function is minimized. That is, we consider the problem

$$\min_{w_i} \|G(\theta)M_S(\theta)\|_\infty \tag{8}$$

subject to

$$\sum_{i=1}^{p} w_i = 1.$$

Here, $G(\theta)$ is a *shaping function* that is used to determine the effort of the H_∞ minimization on typical frequencies. The constraint on the weights is required in order to meet the internal model principle: to have high gain at the harmonics, see also Eq. (7), its numerator should be zero at these frequencies.

To analyze this problem, consider the case where the learning gain $k_r = 1$. In this case, the modifying sensitivity function (7) simplifies to $M_S(\theta) = 1 - W(\theta)e^{-j\theta}$ and the optimization criterion becomes

$$\min_{w_i} \|G(\theta)(1 - W(\theta)e^{-j\theta})\|_\infty,$$

where $W(\theta) = \sum_{i=1}^{p} w_i e^{-(i-1)j\theta}$ is subject to the constraint $\sum_{i=1}^{p} w_i = 1$.

Observe that in this case, for any given G and fixed θ, the function $G(\theta)M_S(\theta)$ is *affine* in the weighting parameters w_i. That is, we can write

$$G(\theta)M_S(\theta) = G(\theta) - \sum_{i=1}^{p} w_i G_i(\theta),$$

where $G_i(\theta) = G(\theta)\mathrm{e}^{-\mathrm{i}j\theta}$. Hence, the optimization amounts to solving

$$\min_{w_i} \sup_{0\leqslant\theta\leqslant 2\pi} \left| G(\theta) - \sum_{i=1}^{p} w_i G_i(\theta) \right|$$

subject to

$$\sum_{i=1}^{p} w_i = 1$$

and its computationally tractable approximation is

$$\min_{w_i} \max_{\theta\in\Theta} \left| G(\theta) - \sum_{i=1}^{p} w_i G_i(\theta) \right|$$

subject to

$$\sum_{i=1}^{p} w_i = 1,$$

where $\Theta = \{\theta_1, \ldots, \theta_K\}$ is a (uniform) finite grid of the interval $[0, \pi]$ (we take half of the period because of symmetry). Equivalently, we wish to solve

$$\min_{w_i,t} \left\{ t \,\middle|\, \left| G(\theta) - \sum_{i=1}^{p} w_i G_i(\theta) \right| \leqslant t, \right.$$
$$\left. \text{for all } \theta \in \Theta, \sum_{i=1}^{p} w_i = 1 \right\} \tag{9}$$

which is a *conic quadratic optimization problem* in the real and imaginary parts of the complex variable

$$z = G(\theta) - \sum_{i=1}^{p} w_i G_i(\theta)$$

by employing that the expression $|z| \leqslant t$ is equivalent to $\sqrt{\mathrm{Re}^2(z) + \mathrm{Im}^2(z)} \leqslant t$.

As an alternative for the conic quadratic optimization, one may convert the latter problem into a linear programming problem by approximating the inequality $|z| \leqslant t$ in (9) by a number, say $2n$, of linear inequalities in the real (a) and imaginary (b) part of the complex number $z = a + bj$. The idea is to inscribe the unit circle in the complex plane by a $2n$ vertex polygon and using that

$$|z| \cos(\phi/2) \leqslant p_n(z) \leqslant |z|,$$

where n is an integer, $\phi = \pi/n$, and

$$p_n(z) := \max_{i=1,\ldots,n} |a \cos(i\phi) + b \sin(i\phi)|$$

is the polyhedral norm consisting of the maximum of absolute values of n linear forms of a and b. The optimization (9) is then effectively approximated within accuracy $1 - \cos(\pi/2n)$ by

$$\min_{w_i,t} \left\{ t \,\middle|\, p_n\left(G(\theta) - \sum_{i=1}^{p} w_i G_i(\theta) \right) \leqslant t, \right.$$
$$\left. \text{for all } \theta \in \Theta, \sum_{i=1}^{p} w_i = 1 \right\}.$$

This is a *linear programming problem*. See, e.g., Ben-Tal and Nemirovski (2001) for details. This can be solved in Matlab using the *linprog* routine, achieving an arbitrary high accuracy of the minimal value of (9).

In the remainder of this section, two cases are distinguished considering the frequency range where low system sensitivity is desirable.

3.1. Robustness for changes in the period time

To achieve robustness for changes in the period time, the magnitude of the modifying sensitivity function is forced to be small in the frequency region close to the periodic frequency, i.e., for frequencies $\theta \in [0, \theta_1]$. This can be attained by choosing the shaping function $G(\theta)$ as

$$G(\theta) = \begin{cases} 1 & \text{if } 0 \leqslant \theta < \theta_1, \\ 0 & \text{if } \theta_1 \leqslant \theta \leqslant \pi. \end{cases}$$

A graphical interpretation is shown in Fig. 3.

With $k_r = 1$, $\theta_1 = \pi/5$, a polyhedral approximation order $n = 2$ and various orders p of the high-order repetitive function W, the following optimal sets of weighting factors

$$W_{\mathrm{opt}}(p, k_r) = (w_1^*, \ldots, w_p^*)$$

for the corresponding linear programming problem are obtained:

$$\begin{aligned} W_{\mathrm{opt}}(2, 1) &= (1.85, -0.85) \approx (2, -1), \\ W_{\mathrm{opt}}(3, 1) &= (2.93, -2.93, 1) \approx (3, -3, 1), \\ W_{\mathrm{opt}}(4, 1) &= (4.01, -6.01, 4.02, -1.02) \approx (4, -6, 4, -1), \\ W_{\mathrm{opt}}(5, 1) &= (4.97, -9.96, 10.09, -5.16, 1.07) \\ &\approx (5, -10, 10, -5, 1). \end{aligned} \tag{10}$$

It can be observed from (10) that the obtained weightings are equal to the analytically derived ones in Steinbuch (2002), see also Singh and Vadali (1993).

The results are shown in Fig. 4. Indeed the sensitivity is made lower near the harmonics (i.e. $\theta_1 = \pi/5$), at the cost of an amplification of the sensitivity at intermediate frequencies.

3.2. Reduction of sensitivity at intermediate frequencies

To improve the system sensitivity at intermediate frequencies, the modifying sensitivity function is minimized over all frequencies: $\theta \in [0, \pi]$. This can be attained by choosing the shaping function $G(\theta) = 1$, for $0 \leqslant \theta \leqslant \pi$. We call this solution the noise robust HORC.

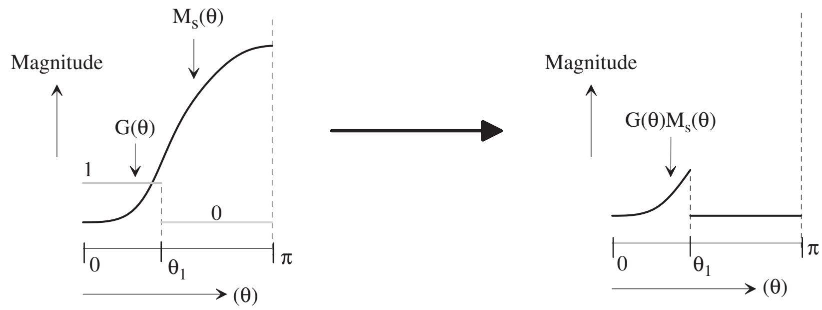

Fig. 3. Minimization of the modifying sensitivity function.

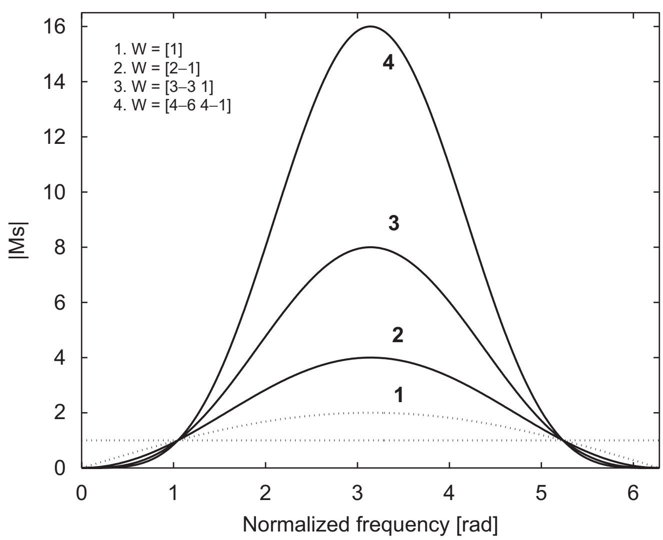

Fig. 4. $|M_{\mathrm{s}}|$ for the period-time robust high order repetitive controller.

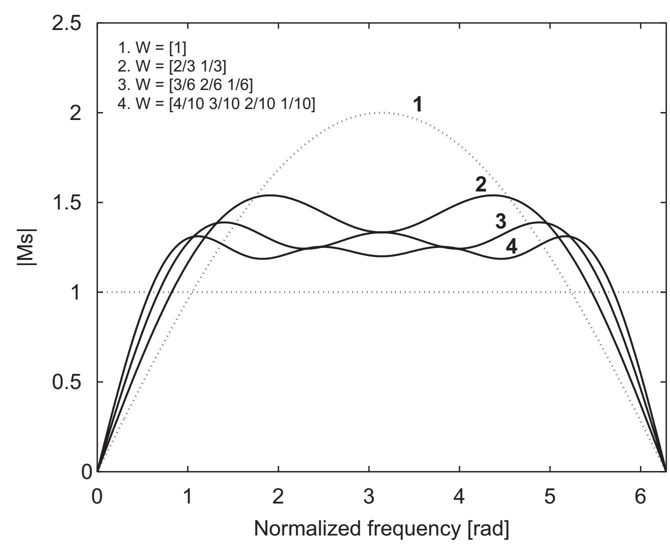

Fig. 5. $|M_{\mathrm{s}}|$ for the noise-robust high-order repetitive controller.

For $k_r = 1$, polyhedral approximation order $n = 2$ and various orders of p, the following optimal sets of weighting factors $W_{\mathrm{opt}}(p, k_r)$ are obtained:

$$\begin{aligned}
W_{\mathrm{opt}}(2,1) &= (0.62, 0.38) \approx \left(\tfrac{2}{3}, \tfrac{1}{3}\right),\\
W_{\mathrm{opt}}(3,1) &= (0.49, 0.33, 0.17) \approx \left(\tfrac{3}{6}, \tfrac{2}{6}, \tfrac{1}{6}\right),\\
W_{\mathrm{opt}}(4,1) &= (0.37, 0.28, 0.22.0.13) \approx \left(\tfrac{4}{10}, \tfrac{3}{10}, \tfrac{2}{10}, \tfrac{1}{10}\right),\\
W_{\mathrm{opt}}(5,1) &= (0.31, 0.25, 0.20, 0.15, 0.09)\\
&\approx \left(\tfrac{5}{15}, \tfrac{4}{15}, \tfrac{3}{15}, \tfrac{2}{15}, \tfrac{1}{15}\right).
\end{aligned} \tag{11}$$

The obtained weights are in correspondence with those obtained in Chang et al. (1995). The results are shown in Fig. 5. Indeed, non-repeatable errors will be less amplified when compared with a standard repetitive controller.

The previously described situations are two extremes. A trade-off between robustness for changes in period time on one hand and sensitivity at intermediate frequencies on the other can be achieved by choosing an appropriate shaping function $G(\theta)$.

4. Application to a compact disc drive

In Fig. 6 a schematic view of a compact disc mechanism is shown. The mechanism is composed of a turn-table DC-motor for the rotation of the compact disc, and a radial arm for track-following. An objective lens, suspended by two parallel leaf springs, can move in a vertical direction to give a focusing action.

The difference between the radial track position and the spot position is detected by the optical pick-up (Stan, 1999); it generates a radial error signal (Steinbuch, van Groos, Schootstra, Wortelboer, & Bosgra, 1998).

The radial actuator frequency response has been measured (under closed-loop conditions) and used to fit a stable sixth order parametric model. The results are plotted in Fig. 7. The low-frequent deviations between measurement and model are due to wrong measurements (low coherence), whereas above 2000 Hz modelling errors occur due to under-modelling. This means that the modelling information used for the design of the learning filters (i.e. based on the complementary sensitivity function) is valid up to a few kHz. We will later see that we restrict learning upto 200 Hz, and not beyond.

The tracking control loop has a cross-over frequency of 600 Hz. The feedback controller is a lead filter with integral action (PID controller Steinbuch & Norg (1998)). The disc is assumed to rotate at a frequency of 12.5 Hz.

4.1. Design of the repetitive controller

A straightforward choice for the learning filter L would be

$$L = k_r T^{-1}, \tag{12}$$

where T is the complementary sensitivity function and k_r the learning gain. However, in many applications, as is the case here, T^{-1} is not proper and also T will be non-minimum phase. As a result, the computation of T^{-1} will lead to a non-proper or unstable L-filter. To overcome this problem, Tomizuka and others (Tomizuka, 1987) developed the so-called ZPETC algorithm, in which the non-minimum phase (or 'unstable') zeros in T are approximated by stable poles in L. This has been applied and in Fig. 8 the frequency response is shown of a proper and stable learning filter L. The present design is therefore not limited to systems in which the complementary sensitivity function is bi-proper or non-minimum phase.

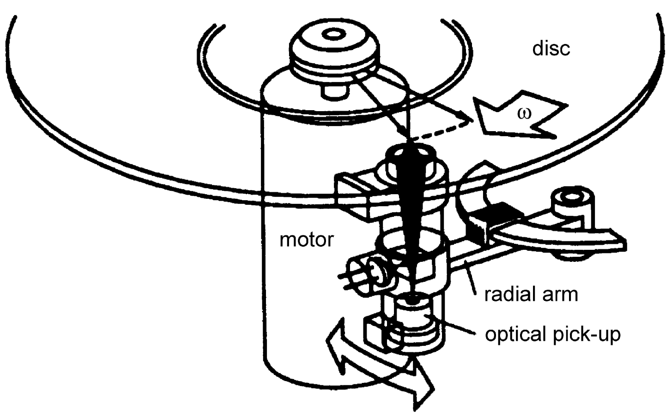

Fig. 6. Schematic view of a rotating arm compact disc mechanism.

As robustness filter Q a symmetric and even order (type I) FIR filter is constructed (low computational complexity). The cut-off frequency of Q is specified at 200 Hz. This implies that up to 16 (200/12.5) harmonics will be suppressed. Raising the bandpass frequency results in disturbance rejection at higher harmonics, but then instability may occur, because L does not match the inverse of T well at high frequencies. Furthermore, the Q-filter order is set to 200. For lower orders the cut-off frequency did not correspond to the desired value. This is probably due to the high sample frequency of the filter ($f_s = 25$ kHz).

The magnitude of the resulting sensitivity function is plotted in Fig. 9, for the system with and without repetitive control. From these figures it can be concluded that the repetitive controller provides a better disturbance rejection at the repetitive frequency and its harmonics at the cost of a degraded performance at intermediate frequencies, as expected.

4.2. Experimental results

When the actual rotation period of the turntable motor is measured, it turns out that the number of revolutions per second is not a constant. The variation is in the range of ± 0.05 Hz.

The measurements are performed for a HORC of order three ($p = 3$), with the following set of weighting factors:

$$\begin{aligned} &W_1 = (0, 0, 0) && \text{standard PID,} \\ &W_2 = (1, 0, 0) && \text{single delay repetitive control,} \\ &W_3 = (3, -3, 1) && \text{period-time-robust HORC,} \\ &W_4 = \left(\tfrac{3}{6}, \tfrac{2}{6}, \tfrac{1}{6}\right) && \text{noise-robust HORC.} \end{aligned} \tag{13}$$

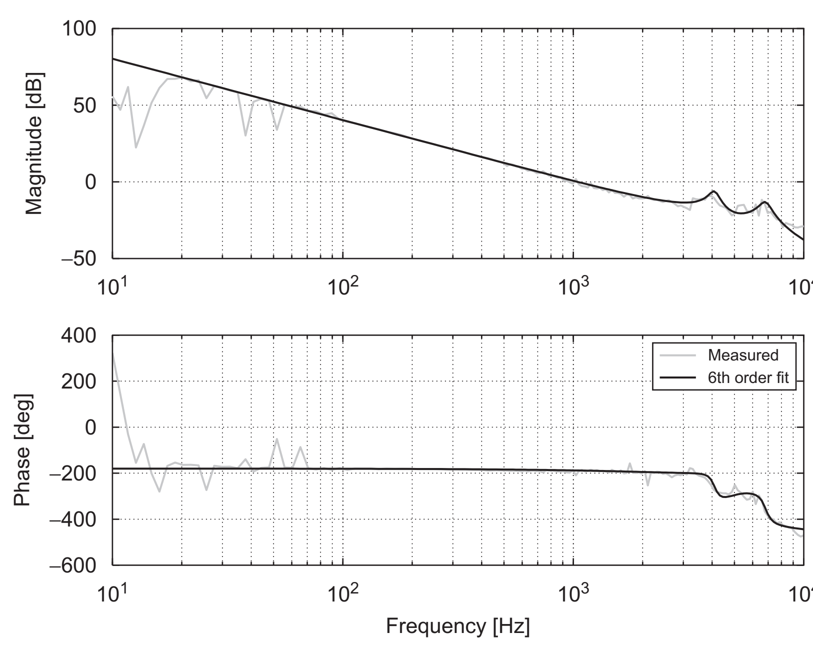

Fig. 7. Frequency response of the CD-player system.

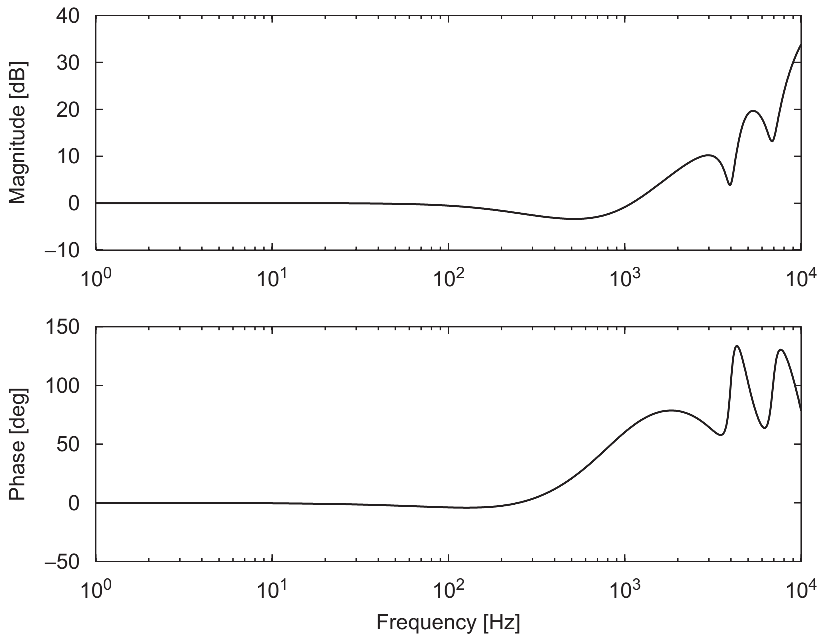

Fig. 8. Bode plot of the learning filter L for $k_r = 1$.

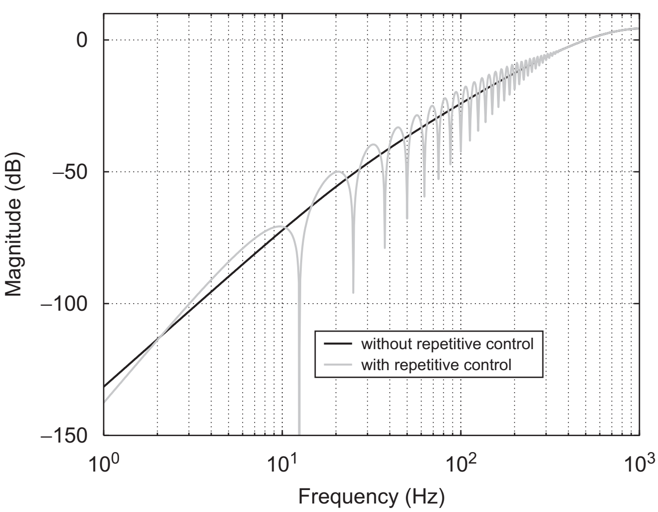

Fig. 9. Sensitivity function of the standard repetitive control system.

In Fig. 10 the measured power spectrum of the error for the different configurations of the repetitive controller is depicted for a rotational frequency of 12.50 Hz. It can be seen that for the system without repetitive controller, W_1, the spectrum of the error has large peaks at the rotational frequency of 12.50 Hz and its harmonics. The noise level between the harmonics is relatively low. This merits the application of the period-robust HORC method. The repetitive controller effectively reduces this peak. But looking at the different configurations, it can be seen that the W_3 configuration almost perfectly reduces the disturbance at the periodic frequency, while for W_2 and W_4 there is still some power left. According to theory and simulations, the reduction at the repetitive frequency should be equal for all repetitive configurations. This difference can be related to the variation in the rotational frequency of the CD-player turntable (± 0.05 Hz). Even small deviations of the disturbance period time from the delay time of repetitive controller result in a degradation of the sensitivity of the W_2 and W_4 configuration, while it does not affect the sensitivity of the W_3 configuration.

Looking in Fig. 10 at the frequency range between two successive harmonics, it can be seen that for the W_2 and W_3 configuration, the system sensitivity at intermediate frequencies is degraded due to the repetitive control action. Here the 'noise proof' repetitive controller, W_4, shows its effectiveness. For this configuration the disturbance rejection between two harmonics is not noticeably deteriorated with respect to the system without repetitive control. These observations are in accordance with the magnitude plots of the modifying sensitivity function of Figs. 4 and 5: the improved robustness for changes in the period time for W_3 goes along with a reduced system sensitivity at intermediate frequencies. The W_4 configuration is less robust for period time changes, but provides better disturbance reduction at intermediate frequencies. With respect to these properties, the single delay configuration lies between both high-order configurations.

5. Conclusions

The use of memory loops is beneficial in systems with repetitive disturbances or tasks. In order to improve the capabilities of repetitive controllers for those cases where the periodicity is hard to measure and is subject to variation and/or where off-harmonics (or noise) occur, the possibility of high-order repetitive control is investigated. A new design algorithm has been developed, which uses simple linear programming techniques to design the repetitive controller. Both a noise-robust and a

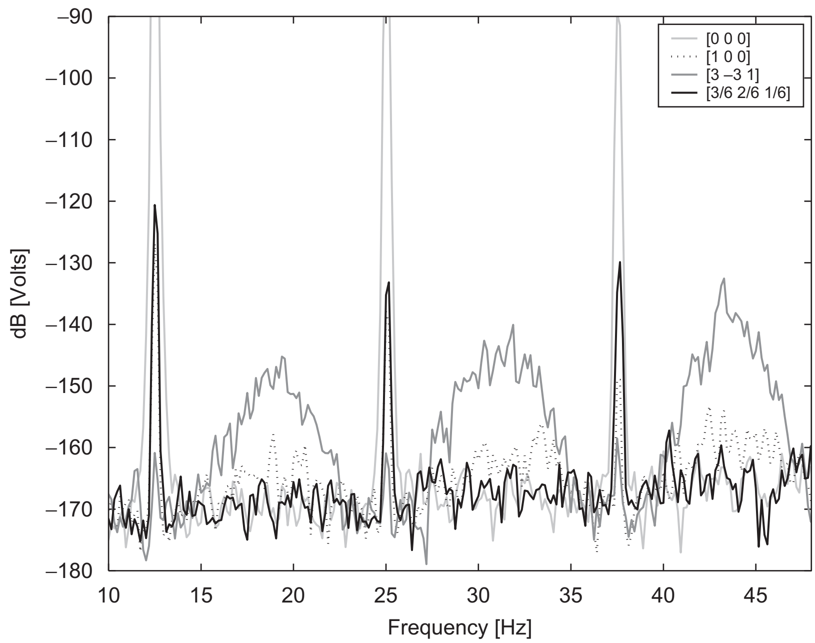

Fig. 10. Power spectrum for a rotational frequency of 12.50 Hz.

period-time-robust high-order repetitive controller have been implemented successfully in a digital control setup of a compact disc player.

The design of high-order repetitive controllers clearly depends on the noise models which can be developed for a certain application. This issue needs further attention, especially in the case where disturbance characteristics are time-varying. In such cases adaptive versions can be designed in which the coefficients of the filters are changing as a function of the signal properties as they are measured on-line. However, stability and convergence issues need further research for such solutions.

Another interesting extension is the use of such high-order structures in iterative learning control, in order to decrease the sensitivity of the solution for the specific trajectories. Also this is subject for further research.

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Maarten Steinbuch is full professor in Systems & Control. He received an M.Sc. degree and a Ph.D. degree from Delft University of Technology, Delft, The Netherlands, in 1984 and 1989, respectively. From 1987 to 1998 he was with Philips Research Labs., Eindhoven. From 1998 to 1999 he was manager of the Dynamics and Control group at Philips Center for Manufacturing Technology. Since 1999 he is full professor of the Control Systems Technology group of the Mechanical Engineering Department of Eindhoven University of Technology. His research interests are modelling, design and control of motion systems and automotive powertrains. He was an associate editor of the IEEE Transactions on Control Systems Technology, of IFAC Control Engineering Practice, and of IEEE Control Systems Magazine. He is currently editor-at-large of the European Journal of Control. In 2003 and 2005 he obtained the 'best teacher' award of the Department of Mechanical Engineering, TU/e. Since July 2006 he is also Scientific Director of the Centre of Competence High Tech Systems of the Federation of Dutch Technical Universities.

Siep Weiland is Associate Professor at the Control Systems Group, Department of Electrical Engineering, Eindhoven University of Technology. He received both his M.Sc. degree (1986) and Ph.D. in mathematics from the University of Groningen in the Netherlands. He was a postdoctoral research associate at the Department of Electrical Engineering and Computer Engineering, Rice University, Houston, USA from 1991 to 1992. Since 1992 he has been affiliated to Eindhoven University of Technology. His research interests are the general theory of systems and control, robust control, model approximation, modeling and control of hybrid systems, identification, and model predictive control. He was Associate Editor of the IEEE Transactions on Automatic Control from 1995 to 1999, of the European Journal of Control from 1999 to 2003, of the International Journal of Robust and Nonlinear Control from 2001 to 2004 and he is currently an Associate Editor for Automatica. He was co-organizer of the 13th IFAC Symposium on System Identification in 2003.

Tarunraj Singh received his B.E. and M.E. degrees and Ph.D. in mechanical engineering from Bangalore University, Indian Institute of Science and the University of Waterloo, respectively. He was a postdoctoral fellow in the Aerospace Engineering Department of Texas A & M University prior to starting his tenure at the University at Buffalo in 1993, where he is currently a Professor in the Department of Mechanical and Aerospace Engineering. He was a von Humboldt fellow and spent his sabbatical at the Technische

Universität Darmstadt in Germany and at the IBM Almaden Research center in 2000–2001. He was a NASA Summer Faculty Fellow at the Goddard Space Flight Center in 2003. His research is supported by the National Science Foundation, AFOSR, NSA, Office of Naval Research and various industries including MOOG Inc. Praxair and Delphi Thermal Systems. His research interests are in robust vibration control, optimal control, nonlinear estimation and intelligent transportation. Dr. Singh has published over 100 refereed journal and conference papers and has presented over 30 invited seminars at various universities and research laboratories.