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On The Structure Equation $F^{p^2}+F=0$

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Abstract: *In this paper, we have studied various properties of the F- sturcture manifold satisfying* $F^{p^2}+F=0$ *where p is odd prime. Metric F-structure, kernel, tangent and normal vectors have also been discussed.*
Keywords: *Differnetiable manifold, complementary projection operators, metric, kernel, tangent and normal vectors.*

I. Introduction

Let M^n be a differentiable manifold of class C^∞ and F be a (1,1) tensor of class C^∞, satisfying

(1.1) $F^{p^2}+F=0$

we define the operators l and m on M^n by

(1.2) $l=-F^{p^2}-1, \qquad m=I+F^{p^2-1}$

where I is the identity operator.
From (1.1) and (1.2), we have

(1.3) $l+m=I, \quad l^2=l, \; m^2=m, \quad lm=ml=0$

$Fl=lF=F, \quad Fm=mF=0,$

Theorem (1.1): Let us define the (1,1) tensors p and q by

(1.4) $p=m+F^{\left(p^2-1\right)/2}, \quad q=m-F^{\left(p^2-1\right)/2},$

Then p and q are invertible operators satisfying

(1.5) $pq=I,\; p^3=q, q^3=p,\; p^2=q^2,\; p^2-p-q+I=0$

Proof: Using (1.2), (1.3) and (1.4)

(1.6) $pq=I, \;\; p^2=q^2=m-l$ etc, the other results follow similarly

Theorem (1.2): Let us define the (1,1) tensors α and β by

(1.7) $\alpha=m+F^{p^2-1}, \beta=m-F^{p^2-1},$ then

(1.8) $\alpha^2=I=\beta$

Proof: Using (1.2), (1.3) and (1.7), we get the results

Theorem (1.3): Let us define the (1,1) tensors γ and δ by

(1.9) $\gamma=l-F^{p^2-1}, \;\; \delta=l+F^{p^2-1},$ then

(1.10) $\gamma^n=2^n l, \;\; \delta=0$

Proof: Using (1.2), (1.3) and (1.9), $\delta=0,$

$$\gamma=2l, \gamma^2=4l=2^2 l \,....\; \gamma^n=2^n l$$

II. Metric F-Structure

For F satisfying (1.1) and Riemannian metric g, let

(2.1) $`F\left(X,Y\right)=g\left(FX,Y\right)$ is skew symmetric then

(2.2) $g(FX,Y)=-g(X,FY)$

and $\{F,g\}$ is called metric F-structure

Theorem (2.1): For F satisfying (1.1)

(2.3) $g\left(F^{(p^2-1)/2}X,F^{(p^2-1)/2}Y\right)=-g(X,Y)+\grave{m}(X,Y)$

where

(2.4) $\grave{m}(X,Y)=g(mX,Y)=g(X,mY)$

Proof: Using (1.2), (2.2), (2.4) and $(p^2-1)/2$ being a multiple of 4, we have

(2.5) $$g\left(F^{(p^2-1)/2}X,F^{(p^2-1)/2}Y\right)=(-1)^{(p^2-1)/2}g\left(X,F^{p^2-1}Y\right)$$
$$=g(X,-lY)$$
$$=-g(X,lY)$$
$$=-g(X,(I-m)Y)$$
$$=-g(X,Y)+\grave{m}(X,Y)$$

III. Kernel, Tangent And Normal Vectors

We define

(3.1) $Ker\ F=\{X:FX=0\}$

(3.2) $Tan\ F=\{X:FX=\lambda X\}$

(3.3) $Nor\ F=\{X:g(X,FY)=0,\forall Y\}$

Theorem (3.1): For *F* satisfying (1.1) we have

(3.4) $Ker\ F=Ker\ F^2=\ldots\qquad=Ker\ F^{p^2}$

(3.5) $Tan\ F=Tan\ F^2=\ldots\qquad=Tan\ F^{p^2}$

(3.6) $Nor\ F=Nor\ F^2=\ldots\qquad=Nor\ F^{p^2}$

Proof: to (3.4) Using (1.1), let $X\in Ker\ F$

$$\Rightarrow FX=0$$
$$\Rightarrow F^2X=0$$
$$\Rightarrow X\in Ker\ F^2$$

Thus

(3.7) $Ker\ F\subseteq Ker\ F^2$

Now let $X\in Ker\ F^2\Rightarrow F^2X=0$

$$\Rightarrow F^3X=0$$

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$$\Rightarrow F^{p^2}X=0$$
$$\Rightarrow FX=0$$

$$\Rightarrow X \in Ker\ F$$

Thus

(3.8) $\quad Ker\ F^2 \subseteq Ker\ F$

From (3.7) and (3.8) we get

(3.9) $\quad Ker\ F \subseteq Ker\ F^2$

proceeding similarly we get (3.4)

Proof to (3.5): Let

$$X \in Tan\ F \Rightarrow FX = \lambda X$$
$$\Rightarrow F^2X = F(\lambda X) = \lambda^2 X$$
$$\Rightarrow X \in Tan\ F^2$$

Thus

(3.10) $\quad Tan\ F \in Tan\ F^2$

Now let

$$X \in Tan\ F^2 \Rightarrow F^2X = \lambda^2 X$$
$$\Rightarrow F^3X = \lambda^3 X$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\Rightarrow F^{p^2}X = \lambda^p X$$
$$\Rightarrow -FX = \lambda^p X$$
$$\Rightarrow FX = -\lambda^p X$$
$$\Rightarrow X \in Tan\ F$$

Thus

(3.11) $\quad Tan\ F^2 \in Tan\ F$

From (3.10) and (3.11)

(3.12) $\quad Tan\ F = Tan\ F^2$ proceeding similarly we get (3.5)

Proof to (3.6): Let

$$X \in Nor\ F \Rightarrow g(X, FY) = 0$$
$$\Rightarrow g(X, F^2Y) = 0$$
$$\Rightarrow X \in Nor\ F^2$$

Thus

(3.13) $\quad Nor\ F \in Nor\ F^2$

Now let

$$X \in Nor\ F^2 \Rightarrow g(X, F^2Y) = 0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\Rightarrow g(X, F^{p^2}Y) = 0$$
$$\Rightarrow g(X, FY) = 0$$
$$\Rightarrow X \in Nor\ F$$

Thus

(3.14) $Nor\ F^2 \in Nor\ F$

From (3.13) and (3.14), we get

(3.15) $Nor\ F = Nor\ F^2,$

Proceeding similarly we get (3.6)

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