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Amplification of High Frequency Wave through Nonlinear Wave Particle Interaction with Drift Wave Turbulence in an Inhomogeneous Plasma

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Abstract: *A theoretical study on the amplification of high frequency wave has been made on the basis of a nonlinear wave particle interaction with drift eave turbulence in an inhomogeneous plasma. The drift wave turbulence, which is one of the common features of inhomogeneous plasma, is taken as the low frequency resonant mode field and is found to be strongly in phase relation with thermal particles. The plasma particles accelerated by drift wave turbulence field, may transfer energy to high frequency ion acoustic wave through a modulated field. A Max wellian distribution function model for inhomogeneous plasma has been considered under the standard local approximation, we have estimated the growth rate of ion acoustic wave, which is obtained by using the nonlinear dispersion relation. It has been found that amplification of ion acoustic wave is possible at the expense of drift turbulence energy.*

I. Introduction

Drift waves and drift instabilities occupy a special place in the spectrum of collective plasma processes. This is because under laboratory conditions gradients in temperature, density, magnetic field and even impurity concentration are inevitable. Whenever a gradient exits, a plasma current or particle drift exists; drift waves are supported by these gradients and instabilities can tap the energy in the drifts. In addition drift modes propagating oblique to the magnetic field can tap the thermal energy of particles streaming along field lines. It has been observed that drift wave turbulence is one of the dominating turbulence wave energy available in any magnetically confined plasma [1]. The universal appearance of a broad band of drift wave fluctuation with frequqncy $4\omega/2\pi \simeq 50 - 600kHz$ at perpendicular wave number 1-15/ cm was reported in torodial confinement device for both the tokamak and helical-stellarator systems [2]. Many fluctuations and transport studies [2] in torodial confinement facilities around the world, including Tokmak Fusion Test Reactor (TFTR), Tora Supra and Advanced Torodial facility were taken in 1980's and 1990's. which referred to these initial findings of the drift wave turbulence and associated radiaal transport. In recent analysis, unsable resistive drift wave has been predicted in the Helimak experiment [3].

The study on the role of drift wave turbulence associated with plasma inhomogeneity for the generation of various unstable radiation is an interesting issue for plasma physicists. The study of the transfer of wave energy from low frequency turbulence to ion acoustic wave has been addressed by a number of authors [4 , 5]. In this paper, we are considering nonlinear wave-particle interaction process in presence of drift wave turbulence, on the baiss of new plasma turbulence theory. The new plasma turbulence theory [6-8] predicted a wave -particle interaction process called plasma maser effect, which makes a system enable for transfer of wave energy from the low frquency mode to the high frequency mode.Plasma-maser interaction is a nonlinear mode - mode coupling process between two types of waves in a turbulent plasma [1]. The first type of plasma wave is the low-frequency resonant mode wave and the second type is the high-frequency non-resonant mode wave. The resonant waves are those for which the Cherenkov resonance condition $\omega - \vec{k} \cdot \vec{v} = 0$ is satisfied, whereas the non-resonant waves are those for which both Cherenkov condition and the nonlinear scattering conditions are not satisfied, i.e. $\Omega - \vec{K}.\vec{v} \neq 0$ and $(\Omega - \omega) - (\vec{K} - \vec{k}) \cdot \vec{v} \neq 0$. Here ω and Ω are frequencies of the resonant and non-resonant waves, respectively, and $\vec{k}$ and $\vec{K}$ are the corresponding wave numbers. As a result of plasma maser effect transfer of the wave energy from low-frequency resonant mode to the high-frequency non-resonant mode may be possible [2]. It has been theoretically established that, plasma maser effect is found to be effective in an open system, where free energy from external sources are available [3] in the form of external magnetic field. Since plasma maser effect is energy up conversion of low-frequency turbulence to high-frequency modes, it plays an important role in space plasma and as an effective radiation mechanism in the magnetosphere plasma.

After the prediction of this new non-linear wave particle interaction process there has been an increasing interest in theoretical, experimental and observational studies [5-8]in plasma-maser effect. This mode-mode coupling process has been applied to auroral kilometric radiation, type III radio emission, chorus related electrostatic bursts, Jovian kilometric radiation and Saturnian kilometric radiation. The transition effects [9]i.e.,effects of transition damping or amplification,which can contribute to the plasma-maser interaction in the case of regular resonant waves are important. The nonlinear evolution of the resonant modes [10] and electron distribution function can be observed in the presence of the non-resonant turbulence. The basic physics involved in the process i.e.,the energy and momentum conservation relations among the waves and the electrons was investigated [11] and it has been shown that the energy and momentum conservation relations between particle kinetic energy and wave energy is satisfied for plasma-maser process, while the Manley-Rowe relation for plasma waves is violated and as a result an efficient energy up-conversion (down-conversion) from the low-frequency resonant mode to high-frequency non-resonant mode becomes possible. The recent advances in the theory of plasma-maser effect have been reviewed by Vladimirov et.al.[12].

In almost all the studies in plasma maser effect have been carried out considering the the plasma system as homogeneous [9-11], recently attempts have been to investigate the role of density gradient parameter in energy up-conversion process through plasma maser effect in inhomogeneous plasma [4,5,12,13,14]. In this paper, we have considered the plasma-maser interaction of ion acoustic wave with the low-frequency drift wave turbulence. Considering a Maxwellian model distribution function under standard local approximation for inhomogeneous plasma [15]. In this study, the growth rate of the test high-frequency ion acoustic wave is obtained with the involvement of spatial density gradient parameter.

II. FORMULATION

We consider a magnetically confined plasma in presence of drift wave turbulence. The ion acoustic mode, present in the system, is considered as super imposing perturbation field to the system.The confining magnetic field with negligibly small gradient is taken along the $\vec{z}$ direction $\vec{B}_0 = B_0(y)$. For such an inhomogeneous system [16] the particle distribution function is considered as

$$f_{0e}(v_\perp, v_\parallel, y + \frac{v_x}{\Omega_e}) \simeq f_{0e}(\vec{v}, y) + \frac{v_x}{\Omega_e}\frac{\partial f_{0e}(\vec{v}, y)}{\partial y}$$
$$\simeq f_{0e}(\vec{v}, y)\left\{1 + \frac{v_x}{\Omega_e}\epsilon'\right\}. \qquad (1)$$

.

where

$$\epsilon' = \left[\frac{1}{f_{0e}}\frac{\partial f_{0e}(\vec{v}, y)}{\partial y}\right]_{y=0}, \qquad (2)$$

is the density gradient for electrons, $f_{0e}(\vec{v}, y)$ is the distribution function for guiding center, $v_x = v_\perp \cos\phi$, $v_\perp$ and $v_\parallel$ are components of velocity along and perpendicular direction of the external magnetic field, ϕ is the phase angle of the particle in the orbit and $\Omega_e = eB_0/mc$ is the electron cyclotron frequency.

The interaction of high-frequency ion acoustic mode wave with low-frequency drift wave turbulence is governed by Vlasov-Poisson equations

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} - \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial}{\partial \vec{v}}\right] F_{0e}(\vec{r}, \vec{v}, t) = 0. \qquad (3)$$

and

$$\vec{\nabla}\cdot\vec{E} = 4\pi e n_e \int f(\vec{r}, \vec{v}, t) dv. \qquad (4)$$

According to the linear response theory of a turbulent plasma [17], the unperturbed electron distribution function and fields are

$$F_{0e} = f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e}, \qquad (5)$$

and

$$\vec{E}_{0e} = \epsilon\vec{E}_l + \epsilon^2\vec{E}_2. \qquad (6)$$

where ϵ is a small parameter associated with low-frequency turbulence fields,$\vec{B}_0 = (0, 0, B_0)$ is the external magnetic field, f_{0e} is space and time averaged part, f_{1e},f_{2e} are fluctuating parts of the distribution function. The wave fields are $\vec{E}_l = (E_{l\perp}, 0, E_{l\parallel})$ with propagating vector $\vec{k} = (k_\perp, 0, k_\parallel)$ and $\vec{B}_l = (0, B_{ly}, 0)$,

To the order of ϵ from Eq.(3), we have

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} - \frac{e}{m}\left(\frac{\vec{v}\times\vec{B}_0}{c}\right)\cdot\frac{\partial}{\partial \vec{v}}\right] f_{1e} = \frac{e}{m}\left(\vec{E}_l + \frac{\vec{v}\times\vec{B}_l}{c}\right)\cdot\frac{\partial}{\partial \vec{v}} f_{oe}. \qquad (7)$$

To find low-frequency electron distribution function perturbation $f_{1e}(\vec{k}, \omega)$, we use Fourier transforms

$$A(\vec{r}, \vec{v}, t) = \sum_{\vec{k},\omega} A(\vec{k}, \omega)\exp[i(\vec{k}\cdot\vec{r} - \omega t)]. \qquad (8)$$

and follow the method of characteristics [15],we have

$$f_{1e}(\vec{k}, \omega) = \left(\frac{ie}{m}\right)\frac{E_{l\parallel}(\vec{k}, \omega)\frac{\partial}{\partial v_\parallel} f_{0e}}{\omega - k_\parallel v_\parallel + i0}. \qquad (9)$$

We now perturb the quasisteady state by a high frequency-test ion acoustic mode wave field $\mu\delta\vec{E}_h$ with a propagating vector $\vec{K} = (K_\perp 0, 0)$, electric field $\delta\vec{E} = (\delta E_h, 0, 0)$ and a frequency Ω . Thus the total perturbed electric field, magnetic field and the electron distribution function are

$$\delta\vec{E} = \mu\delta\vec{E}_h + \mu\epsilon\delta\vec{E}_{lh} + \mu\epsilon^2\Delta\vec{E} \tag{10}$$

$$\delta\vec{B} = 0 + \mu\epsilon\delta\vec{B}_{lh}$$

$$\delta f = \mu\delta f_h + \mu\epsilon\delta f_{lh} + \mu\epsilon^2\Delta f.$$

where $\delta\vec{E}_{lh}$, $\Delta\vec{E}$ are the modulated electric fields, δf_h is the fluctuating part of electron distribution function due to perturbation of high-frequency ion acoustic mode wave and δf_{lh}, Δf are electron distribution functions corresponding to modulated fields.

Linearising the Vlasov Eq.(3) to the order $\mu, \mu\epsilon, \mu\epsilon^2$, we obtain

$$P\delta f_h = \frac{e}{m}\delta\vec{E}_h \cdot \frac{\partial}{\partial\vec{v}} f_{0e}, \tag{11}$$

$$P\delta f_{lh} = \frac{e}{m}\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}} f_{oe} + \frac{e}{m}\left(\vec{E}_l + \frac{\vec{v}\times\vec{B}_l}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\delta f_h + \frac{e}{m}\left(\delta\vec{E}_h\cdot\frac{\partial}{\partial\vec{v}} f_{1e}\right). \tag{12}$$

.

$$P\Delta f = \frac{e}{m}\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}} f_{1e} + \frac{e}{m}\left(\vec{E}_l + \frac{\vec{v}\times\vec{B}_l}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\delta f_{lh} \tag{13}$$

where the operator P is given by

$$P = \frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial\vec{r}} - \frac{e}{m}\left(\frac{\vec{v}\times\vec{B}_0}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}.$$

we have omitted the second order field quantities which can be justified under random phase approximations.

Using the Fourier transform (Eq. (8)) and integrating along the unperturbed orbit, we evaluate the various perturbed distribution functions.

The modulated electric field δE_{lh} is obtained by using Ampere's equations

$$\nabla\times\delta\vec{B} = \frac{1}{c}\frac{\partial\delta\vec{E}_{lh}}{\partial t} + \frac{4\pi}{c}\vec{J}.$$

$$\vec{J} = -en \int \vec{v} \sum_j f_{lhj}(\vec{K} - \vec{k}) d\vec{v}.$$

as

$$\delta E_{lh} = \frac{4\pi e n_e(\Omega - \omega)}{(\Omega - \omega)^2 - c^2 k_\perp^2} \int v_\| \sum_j \delta f_{lhj}(\vec{K} - \vec{k}) d\vec{v}$$
$$= -\frac{4\pi e^2 n_e(\Omega - \omega)}{mR[(\Omega - \omega)^2 - c^2 k_\perp^2]} \frac{\delta E_h}{K_\perp} \int v_\| \left[\left\{ \frac{ie}{m} \left(1 + \left\{ \Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e} \right\} \right. \right. \right.$$
$$\left. \left. \sum_{a,b} \frac{J_a(\alpha) J_b(\alpha) \exp\{i(b-a)\theta\}}{a\Omega_e - \Omega} \right) \right\} \times \left[\left\{ \left(1 - \frac{k_\| v_\|}{\omega} \right) E_{l\perp} + \right. \right.$$
$$\left. \frac{k_\perp v_\|}{\omega} E_{l\|} \right\} \left(\frac{m}{T_e K_\perp} \right) \left(1 + \left\{ \Omega - \omega + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e} \right\} \times \right.$$
$$\left. \sum_{p,q} \frac{J_p(\alpha') J_q(\alpha') \exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\| v_\| + \omega} \right) f_{0e} - \left\{ \left(1 - \frac{k_\| v_\perp}{\omega} \right) E_{l\perp} + \right.$$
$$\left. \left. \frac{k_\perp v_\|}{\omega} E_{l\|} \right\} \frac{\partial f_{0e}}{\partial v_\|} \sum_{s,t} \frac{(2s/\alpha') J_s(\alpha') J_t(\alpha') \exp[i(t-s)\theta]}{s\Omega_e - \Omega - k_\| v_\| + \omega} \right] +$$
$$\frac{m}{T_e} \left(\frac{ie}{m} \frac{E_{l\|}(\vec{k}, \omega) \frac{\partial}{\partial v_\|} f_{oe}}{\omega - k_\| v_\| + i0} \right) \left(1 + \left\{ \Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e} \right\} \times \right.$$
$$\left. \left. \sum_{a,b} \frac{J_a(\alpha) J_b(\alpha) \exp[i(b-a)\theta]}{a\Omega_e - \Omega} \right) \right] d\vec{v}. \tag{14}$$

.

where $\alpha = K_\perp v_\perp / \Omega_e$ and $\alpha' = (K_\perp - k_\perp) v_\perp / \Omega_e$ and

$$R = 1 + \frac{4\pi e^2 n_e(\Omega - \omega)}{(\Omega - \omega)^2 - c^2 k_\perp^2} \int v_\| \left[\left(1 - \frac{k_\| v_\|}{\omega} \right) \frac{ie}{m} (1 + \{\Omega - \right.$$
$$\omega + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e} \Big\} \sum_{a,b} \frac{J_a(\alpha') J_b(\alpha') \exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega} \Big) f_{0e} + \tag{15}$$
$$\left. \frac{k_\| v_\perp}{\Omega - \omega} \frac{\partial f_{0e}}{\partial v_\|} \sum_{a,b} \frac{(2a/\alpha') J_a(\alpha') J_b(\alpha') \exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega} \right] d\vec{v}.$$

.

The nonlinear dielectric function $\epsilon_h(\vec{K},\Omega)$ of the ion acoustic mode wave with frequency Ω, in the presence of the drift wave turbulence is obtained, from Poisson equation, which can be expressed as

$$\epsilon_h(\vec{K},\Omega) = \epsilon_0(\vec{K},\Omega) + \epsilon_d(\vec{K},\Omega) + \epsilon_p(\vec{K},\Omega). \tag{16}$$

where $\epsilon_0(\vec{K},\Omega)$ is the linear part, $\epsilon_d(\vec{K},\Omega)$ is the direct coupling part and $\epsilon_p(\vec{K},\Omega)$ is the polarization coupling part. Their expressions are given by

$$\begin{aligned}\epsilon_0(\vec{K},\Omega) = 1 + \left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{m}{T_e}\right)\int\Bigg\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \\ \times \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega}\Bigg\}\, d\vec{v}.\end{aligned} \tag{17}$$

The expression for $\epsilon_d(\vec{K},\Omega)$ is bulky. So we write the expression of the dominant part only as follows

$$\begin{aligned}\epsilon_d(\vec{K},\Omega) = -\frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{e}{m}\right)^2 |E_{l\parallel}(\vec{k},\omega)|^2 \int\Bigg[\frac{k_\perp v_\parallel}{\omega}\Bigg\{1 + \Big(\Omega - \\ -\frac{\epsilon' T_e K_\perp}{m\Omega_e}\Big)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigg\} \times \\ \Bigg[\Bigg\{\frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e K_\perp}\Bigg(1 + \left\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\} \times \\ \sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg) - \left(1 + \frac{k_\perp v_\perp}{\omega}\right) \times \\ \frac{\partial}{\partial v_\parallel}\sum_{s,t}\frac{(2s/\alpha')J_s(\alpha')J_t(\alpha')\exp[i(t-s)\theta]}{s\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg\} f_{0e} + \\ \frac{m}{T_e}\left(\frac{\frac{\partial}{\partial v_\parallel}f_{oe}}{\omega - k_\parallel v_\parallel + i0}\right)\Bigg\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \times \\ \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigg\}\Bigg] - \\ \frac{k_\perp v_\perp}{\omega}\sum_{a,b}\frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\frac{\partial}{\partial v_\parallel} \times \\ \Bigg\{\Bigg(1 + \left\{\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigg) \times \\ \Bigg[\frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e}\Bigg(1 + \left\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\} \times \\ \sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg)\Bigg\} f_{0e} - \left(1 + \frac{k_\parallel v_\perp}{\omega}\right) \times \\ \frac{\partial f_{0e}}{\partial v_\parallel}\sum_{s,t}\frac{J_s(\alpha')J_t(\alpha')\exp[i(t-s)\theta]}{s\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg] + \\ \frac{m}{T_e}\left(\frac{\frac{\partial}{\partial v_\parallel}f_{oe}}{\omega - k_\parallel v_\parallel + i0}\right)\left(1 + \left\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\} \times\right.\end{aligned}$$

$$\sum_{p,q} \frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigr)\Bigr\}\Bigr]\, d\vec{v}. \qquad (18)$$

The expression for the polarization coupling term is even more bulky , taking the dominant contribution term only

$$\epsilon_p(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{P[(\Omega-\omega)^2 - c^2k_\perp^2]}|E_{l\parallel}(\vec{k},\omega)|^2\,[A+B]\,. \qquad (19)$$

where A and B are given by

$$A = \int v_\parallel \Bigl[\Bigl\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigr\} \times$$

$$\Bigl\{\frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e K_\perp}\Bigl(1 + \Bigl\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigr\} \times$$

$$\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigr) f_{0e} - \frac{k_\perp v_\parallel}{\omega}\frac{\partial}{\partial v_\parallel} f_{0e} \times$$

$$\sum_{s,t}\frac{(2s/\alpha')J_s(\alpha')J_t(\alpha')\exp[i(t-s)\theta]}{s\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigr\} + \frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e} \times$$

$$\Bigl\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigr\} \times$$

$$\left(\frac{i\epsilon}{m}\frac{\frac{\partial}{\partial v_\parallel f_{0e}}}{\omega - k_\parallel v_\parallel + i0}\right)\Bigr]\Bigl\{\Bigl(\Bigl(1 - \frac{k_\parallel v_\parallel}{\omega}\Bigr)\frac{m}{T_e}(1 + \{\Omega - \omega + k_\parallel v_\parallel$$

$$\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigr)\Bigr\}\Bigr]\, d\vec{v}. \qquad (18)$$

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$$\epsilon_p(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{P[(\Omega-\omega)^2 - c^2k_\perp^2]}|E_{l\parallel}(\vec{k},\omega)|^2\,[A+B]\,. \qquad (19)$$

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$$A = \int v_\parallel \Bigl[\Bigl\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigr\} \times$$

$$\Bigl\{\frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e K_\perp}\Bigl(1 + \Bigl\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigr\} \times$$

$$\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigr) f_{0e} - \frac{k_\perp v_\parallel}{\omega}\frac{\partial}{\partial v_\parallel} f_{0e} \times$$

$$\sum_{s,t}\frac{(2s/\alpha')J_s(\alpha')J_t(\alpha')\exp[i(t-s)\theta]}{s\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigr\} + \frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e} \times$$

$$\Bigl\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigr\} \times$$

$$\left(\frac{i\epsilon}{m}\frac{\frac{\partial}{\partial v_\parallel f_{0e}}}{\omega - k_\parallel v_\parallel + i0}\right)\Bigr]\Bigl\{\Bigl(\Bigl(1 - \frac{k_\parallel v_\parallel}{\omega}\Bigr)\frac{m}{T_e}(1 + \{\Omega - \omega + k_\parallel v_\parallel$$

$$-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigg\} \sum_{a,b} \frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg) + \frac{k_\parallel v_\perp}{\Omega - \omega}$$

$$\frac{\partial}{\partial v_\parallel} f_{0e} \sum_{a,b} \frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg\} d\vec{v}. \tag{20}$$

and

$$B = \int v_\parallel \Bigg[\Bigg\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \sum_{a,b} \frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigg\} \times$$

$$\Bigg\{\frac{k_\perp v_\parallel}{\omega}\frac{m}{T_e}\Bigg(1 + \Bigg\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigg\} \times$$

$$\sum_{p,q} \frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg) f_{0e} + \frac{k_\perp v_\perp}{\omega}\frac{\partial}{\partial v_\parallel} f_{0e}$$

$$\sum_{s,t} \frac{(2s/\alpha')J_s(\alpha')J_t(\alpha')\exp[i(t-s)\theta]}{s\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg\} + \frac{k_\perp v_\perp}{\omega}\frac{m}{T_e} \times$$

$$\left(\frac{i\epsilon}{m}\frac{\frac{\partial}{\partial v_\parallel f_{0e}}}{-\omega + k_\parallel v_\parallel + i0}\right)\Bigg\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \times$$

$$\sum_{a,b} \frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\Bigg\}\Bigg] \times$$

$$\Bigg\{\sum_{a,b} \frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\frac{\partial}{\partial v_\parallel} \times$$

$$\Bigg\{\left(1 - \frac{k_\parallel v_\parallel}{\Omega - \omega}\right)\frac{m}{T_e}\Bigg(1 + \Bigg\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigg\}$$

$$\sum_{a,b} \frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg)\Bigg\} f_{0e} +$$

$$\sum_{a,b} \frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\frac{\partial}{\partial v_\parallel} \times \tag{21}$$

$$\left(\frac{k_\parallel v_\parallel}{\Omega - \omega}\frac{\partial f_{0e}}{\partial v_\parallel}\sum_{a,b} \frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right)\Bigg\} d\vec{v}.$$

III. Growth Rate

The growth rate of ion acoustic mode wave is calculated by using the following formula:

$$\frac{\gamma_h}{\Omega} = -\left[\frac{\left(\frac{I_m \epsilon_h}{\partial \Omega \partial t}\right)}{\Omega\frac{\partial \epsilon_0}{\partial \Omega}}\right]_{\Omega_r}. \tag{22}$$

We consider the plasma-maser interaction between ion acoustic mode wave and drift wave turbulence. The condition for the plasma-maser is $\omega = k_\parallel v_\parallel$ and assuming $\Omega < K v_\parallel$, we first estimate the linear part of the dielectric function of ion acoustic mode wave from Eq.(17),and considering the fact that for ion acoustic mode wave the most dominant contribution to Bessel's sum comes from the term $a = s = p = 1$,

$$\epsilon_0(\vec{K}, \Omega) = 1 + \left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{m}{T_e}\right) + \left(\frac{\omega_{pe}^2}{2\Omega_e^2}\right)\left(\frac{1}{\Omega_e - \Omega}\right)\left\{\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e \Omega}\right\}. \tag{23}$$

From Eq.(23),we obtain,

$$\frac{\partial\epsilon_0}{\partial\Omega} = \left(\frac{\omega_{pe}^2}{K_\perp^2}\right) \times \frac{1}{(\Omega_e - \Omega)^2} \times \left(\Omega - \frac{e'T_e K_\perp}{m\Omega_e}\right). \tag{24}$$

Since the instabilities comes from the imaginary part of the polarization coupling term, so we calculate only the imaginary part of the polarization coupling term from Eq.(17) as

$$I_m\epsilon_p(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{P[(\Omega-\omega)^2 - c^2k_\perp^2]} \times |E_{l\parallel}(\vec{k},\omega)|^2 \times I_m[A+B]$$

Now, we calculate the imaginary parts of A and B from eqns. (20) and (21) and put in above Eq., we get

$$\begin{aligned} I_m\epsilon_p(\vec{K},\Omega) = &\left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{P[(\Omega-\omega)^2 - c^2k_\perp^2]}|E_{l\parallel}(\vec{k})|^2 \times \\ &\left[\left\{1 + \frac{1}{\Omega_e-\Omega} \times \left(\Omega - \frac{e'T_eK_\perp}{m\Omega_e}\right)\left(\frac{K_\perp^2}{2\Omega_e^2} \times \frac{T_e}{m} + \right.\right.\right. \\ &\left.\left.\frac{(K_\perp-k_\perp)^2}{2(\Omega_e-\Omega)^2} \times \frac{T_e}{m}\right) + \frac{1}{(\Omega_e-\Omega)^2} \times \left(\Omega - \frac{e'T_eK_\perp}{m\Omega_e}\right)^2 \Lambda_1\right\} \times \\ &\left(\frac{k_\perp}{\omega} - \frac{k_\parallel k_\perp}{\omega^2}\right) v_A + \frac{k_\parallel k_\perp}{\omega(\Omega-\omega)} \times \frac{m}{T_e} \times \frac{1}{\Omega_e-\Omega} \times \\ &\left\{\frac{(K_\perp-k_\perp)^2}{2\Omega_e^2} \times \frac{T_e}{m} + \frac{1}{\Omega_e-\Omega} \times \left(\Omega - \frac{e'T_eK_\perp}{m\Omega_e}\right)\Lambda_1\right\} v_A - \\ &\left.\frac{k_\parallel k_\perp}{\omega^2} \times \left(\frac{m}{T_e}\right)^2 \times \left(\frac{2\Lambda_1}{\Omega_e-\Omega}\right)\right] \frac{2\sqrt{\pi}}{v_e^2}\frac{v_A}{k_\parallel} \exp\left\{-\left(\frac{v_A}{v_e}\right)^2\right\}. \end{aligned}$$

where $v_A = \omega/k_\parallel$ is the Alfven velocity and

$$\Lambda_1 = \int_0^\infty 2\pi v_\perp^2 J_1^2(\alpha) J_1^2(\alpha') f_{0e}(v_\perp) dv_\perp.$$

In the similar way we can estimate $P(\vec{K}-\vec{k},\Omega-\omega)$ by expanding , from Eq.(15), about the small argument $\vec{k}$ and ω in R and we have used the relation $\epsilon(\vec{K},\Omega) = 0$. To the lowest order approximation for $\Omega > \omega$,

$$\frac{1}{P(\vec{K}-\vec{k})[(\Omega-\omega)^2 - c^2k_\perp^2]} \simeq \frac{1}{c^2k_\perp^2}$$

The polarization coupling term is the dominating term in the plasma maser effect , so we calculate the growth rate due to the polarization coupling term only by using the formula (Eq.(24)),

$$\frac{\gamma_h}{\Omega} = 16\pi^{3/2}\omega_{pe}^2\Lambda_1 \sum_{\vec{k},\omega} W_T \frac{1}{(Q+1)^2}\left(\frac{k_\parallel}{k_\perp}\right)^2\left(\frac{v_A}{c}\right)^4 \times \left(\Omega - \frac{e'T_eK_\perp}{m\Omega_e}\right)\exp\left\{-\left(\frac{v_A}{v_e}\right)^2\right\} \tag{26}$$

where the normalized turbulence energy of Alfven wave (W_T) is given by

$$W_T = \sum_{\vec{k},\omega} \frac{|E_{l\parallel}(\vec{k},\omega)|}{16\pi NT_e}\left(\frac{k_\perp}{k_\parallel}\right)^2\left(\frac{c}{v_A}\right)^2(Q+1)^2.$$

Here Q is related to the amplitude ratio of electric fild components of the Kinetic Alfven wave

$$\frac{E_{l\parallel}(\vec{k},\omega)}{E_{l\perp}(\vec{k},\omega)} = -\left(\frac{k_\parallel}{k_\perp}\right)\left(\frac{T_e}{T_i}\right)[1 - I_0(\beta_i)\exp(-\beta_i)] \equiv \left(\frac{k_\parallel}{k_\perp}\right)Q^{-1}.$$

here T_e, T_i and I_0 are electron , ion temperatures and modified Bessel function respectively, $\beta_i = \left(\frac{k_\perp}{\rho_i}\right)^2/2$ and ρ_i. is ion gyro radius.

IV. Discussion

In an earlier investigation [18] on interaction among drift wave and high frequency Trivelpiece-Gould mode through plasma maser effect, interesting result about the stability of drift mode in homogeneous plasma were predicted. In this paper, it has been shown that nonlinear damping of drift mode occurs through energy exchange among drift mode and electrostatic Trivelpiece-Gould mode. Here the nonlinear dispersion relation of ion acoustic mode in presence of drift wave turbulence field is evaluated, which is a common feature of inhomogeneous plasma.

Historically, the plasma-maser from the direct coupling term $\epsilon_d(\vec{k},\Omega)$ was pointed by Tystovich and co-worker [19] for an ion-sound turbulence in unmagnetized plasma.But in a closed system the plasma maser contribution from direct coupling part exactly cancels out with the reverse absorption effect [20], i.e.

$$I_m\epsilon_d(\vec{K},\Omega)+\frac{1}{2}\frac{\partial^2\epsilon_0(\vec{K},\Omega,f_{0e}(t))}{\partial\Omega\partial t}=0.$$

In this system the low-frequency turbulence and the background electron distribution function are not fixed by external agents, but are free to evolve self-consistently to form a quasilinear plateau. Then both the plasma maser and the reverse process due to quasilinear effect coexist and there is no net growth of the nonresonant high-frequency test wave.

In an open system, with a particle supply from outside, the electron distribution function is fixed by external agents and the reversed absorption effect vanishes

$$\partial^2\epsilon_0(\vec{K},\Omega,f_{0e}(t))/(\partial\Omega\partial t)=0$$

; then the stationary state without quasilinear plateau is possible. Accordingly, the energy transferred from the low-frequency waves by resonant interaction must go into an unstable high-frequency mode. This type of enhanced radiation process is termed as dissipative structure in plasma turbulence [20]. The dissipation is due to resonant wave-particle collisionless heating. The nonresonant wave amplification or absorption can be due to perturbation of resonant wave-particle collisionless interaction.

Here, an attempt has been made to consider the plasma maser effect among electrons, resonant mode drift wave and the nonresonant ion acoustic mode wave in inhomogeneous magnetized plasma.The inhomogeneity feature of the plasma considered here provides an additional source of free energy to the system and is now important to clarify the role of free energy of these drifting particles in this particular mode-mode coupling process of turbulent inhomogeneous plasma. The nonlinear dispersion relation Eq.(16) is obtained by neglecting the gradients of the confining magnetic field . Since polarization coupling is the dominating term in the Plasma maser effect, the growth rate of ion acoustic mode wave is estimated from polarization coupling term only.

When there is no density gradient ($\epsilon' = 0$) the growth rate of ion acoustic mode for homogeneous plasma is obtained, from Eq.(26), as

$$\frac{\gamma_p}{\Omega} = 16\pi^{3/2}\Lambda_1\omega_{pe}^2 \sum_{\bar{k},\omega} W_T \left(\frac{k_\parallel}{k_\perp}\right)^2 \left(\frac{v_A}{c}\right)^4 (Q+1)^{-2} \qquad (27)$$

When high density gradient is present in the system of plasma then the growth rate of Bernstein mode wave is obtained, from Eq. (26),as

$$\frac{\gamma_p}{\Omega} = 16\pi^{3/2}\Lambda_1 \left(\frac{\omega_{pe}^2}{V_e\Omega_e}\right) \sum_{\bar{k},\omega} W_T \left(\frac{k_\parallel}{k_\perp}\right)^2 \left(\frac{v_A}{c}\right)^4 (Q+1)^{-2}\epsilon' \qquad (28)$$

However for small order gradient there is no effect in the growth rate and Eq.(27) will give the estimate for such small gradient situation.

By using the following observational data in space : $\Omega_e \sim \omega_{pe} \sim 5.6 \times 10^4 rad/sec$, $\Lambda_1 \sim 1$, $v_e \sim 10^{-1}$, $k_\parallel/k_\perp \sim 10^{-1}$ $Q \sim 1.5$ and $v_A/c \sim 10^{-2}$

We have from Eq.(27)

$$\frac{\gamma_p}{\Omega} \sim 10^{-1} W_T \qquad (29)$$

and from Eq. (28)

$$\frac{\gamma_p}{\Omega} \sim 10^{-4} W_T \times \epsilon' \quad (30)$$

and from Eq.(30),

$$\frac{\gamma_p}{\Omega} \sim 10^{-3} W_T \quad (27)$$

taking $\epsilon' = 10$

Thus we may conclude that the density gradients influence the growth rate of ion acoustic mode wave. As the gradients in the confining field are weak in most cases of space and laboratory plasma [21], here in this study, growth rate of ion acoustic mode wave through plasma-maser is calculated by ignoring the gradients in magnetic field.

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