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Probabilistic interval reliability of structural systems

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Abstract

The probabilistic reliability approach is the most widely used method for reliability analysis. The recent research shows that the reliabilities of structural systems strongly depend on the parameters of the probability model. It is possible that the little error in the estimation of the parameters may lead to the remarkable error of the resulting probability. In this study, we introduce the interval approach into the conventional reliability theory. We present a novel approach which allows us to obtain the system failure probability interval from the statistical parameter intervals of the basic variables. This approach is a combination of the two techniques, namely the classical reliability theory and the interval analysis. In the end of this paper, we show the feasibility of the proposed approach through two examples of the truss systems.

Keywords: Probability; Interval analysis; Structural systems; Interval reliability

1. Introduction

Structural reliability analysis for systems plays an important role in the analysis and design of structures. The main purpose of structural reliability analysis is to evaluate the structural probability of structural survival, or its complement, the probability of structural failure, taking into account the uncertainties involved in the problem. The uncertainties are associated with physical quantities such as external loads, associated with geometrical properties such as size of a structural member, and associated with theoretical model used to predict external loads.

In their monograph, Gurov and Utkin (1999) addressed the following fact: “Due to the complexity of systems the information about the functioning of its components has different sources. Part of the information is obtained as a result of statistical experiments and has a probabilistic character. Another part is obtained by the estimation of the experts and in most cases has an interval character. Information can be obtained as a result of non-large amount of observations, prechiding construction of exact probabilistic estimates. Therefore, the development of rigorous mathematical methods of combining the existing information for obtaining

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general estimates of the reliability of the entire system represents an actual problem." (Utkin et al., 1997). Such a combination of the probabilistic and non-probabilistic convex analysis was performed by Elishakoff and Li (1999) to calculate the interval information (continuation) of the reliability function. The hybrid approach was pioneered by Elishakoff and Colombi (1993, 1994). Monograph by Elishakoff et al. (1994) contained the application of this hybrid approach to space shuttle systems. The combined approach was utilized in order to calculate the mean values of the structure's displacements. The contrast of probabilistic and non-probabilistic approaches was conducted by Wu et al. (1990) and Elishakoff et al. (1994a,b). In a series of studies, Qiu et al. (2001a,b, 2004) utilized non-probabilistic convex models in various contexts. Pantelides and Ganzerli (2001) compared fuzzy sets and convex models. Fang et al. (1998) combined fuzzy sets-based method and convex method. Mullen and Muhanna (1999) as well as Rao and Berke (1997) utilized interval methods to find the bounds of structural responses.

In this study, we combine non-probabilistic interval analysis and probabilistic methodology, to determine the bounds of system's structural reliability by utilizing the method developed by Thoft-Christensen and Murotsu (1986) for purely probabilistic treatments.

2. System structural reliability methods

Reliability analysis is to analytically formulate the failure given a failure criteria or failure mode. As a simplification, it is assumed that all states of the structure are divided into two states: fail state and safe state. A function $g(\mathbf{X})$, called limit state function or failure function, is defined such that if $g(\mathbf{X}) > 0$ the structure is in the safe state, and if $g(\mathbf{X}) < 0$ the structure is in the fail state. Given a limit state function $g(\mathbf{X})$ and a joint density function $f_X(\mathbf{x})$ of the random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, the probability of failure is computed by:

$$P_F = \int_{g(\mathbf{x})\leqslant 0} f_X(\mathbf{x})\mathrm{d}x \tag{1}$$

The reliability or probability of survival P_S, is the complement of P_F is computed as $(1 - P_F)$. In this paper, we will use the FORM (First Order Reliability Method) to compute the multi-dimensional integral given by Eq. (1). The method can be divided into three steps. In the first step, the vector of basic variables $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ is transformed into an independent standard (zero mean and unit standard deviation) normal vector $\mathbf{U} = (U_1, U_2, \ldots, U_n)$ using a probability preserving transformation. In the second step, the failure surface in the u-space is approximated as a linear surface. In the final step, the probability content of the u-space can be exactly computed for the linear domain.

Before proceeding to the next section, we will discuss the two types of structure systems first, namely 'parallel systems and series systems.

2.1. Parallel system

The probability of failure of a parallel system with n components described by failure function g_i, is given by

$$P_{Fp} = P\left(\bigcap_{i=1}^{n} g_i(\mathbf{u}) \leqslant 0\right) \tag{2}$$

Using FORM, after linearization, each component g_i can be written as

$$g_i(\mathbf{u}) \approx \boldsymbol{\alpha}_i^T\mathbf{u} + \beta_i \tag{3}$$

Defining $Z_i = \boldsymbol{\alpha}_i^T\mathbf{u}$, where Z_i is a standard normal variable, P_{Fp} can be approximated as

$$P_{Fp} \approx P\left(\bigcap_{i=1}^{n} Z_i \leqslant -\beta_i\right) = \Phi_n(-\boldsymbol{\beta}, \boldsymbol{\rho}) \tag{4}$$

where Φ_n is the n-dimensional standard normal cumulative distribution function, $\boldsymbol{\beta}$ is the vector of β_i's, and $\boldsymbol{\rho}$ is the matrix of correlation coefficients ρ_{ij} defined by

$$\rho_{ij} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j \tag{5}$$

The evaluation of the failure probability is then reduced to the evaluation of the multi-normal integral.

2.2. Series system

The probability of failure of a series system with n components described by failure function g_i, is given by

$$P_{Fs} = P\left(\bigcup_{i=1}^{n} g_i(\mathbf{u}) \leqslant 0\right) \tag{6}$$

The procedure to estimate the probability of failure of series system is similar to the one described for the parallel system. The probability of the union of n components is expressed in terms of the intersection of the complementary events as shown below

$$P_{Fs} = 1 - P\left(\bigcap_{i=1}^{n} g_i(\mathbf{u}) > 0\right) \tag{7}$$

Each failure surface $g_i(\mathbf{u})$ is approximated by a hyperplane (3) using FORM. P_{F_S} is then computed by the multi-normal cumulative distribution function

$$P_{Fs} \approx 1 - \Phi_n(\boldsymbol{\beta}, \boldsymbol{\rho}) \tag{8}$$

3. Failure probability interval of parallel systems and series systems

Based on interval mathematics, the failure probability interval of the parallel system and series system can be obtained from Eqs. (4) and (7)

$$P_{Fp}^I \approx P\left(\bigcap_{i=1}^{n} Z_i^I \leqslant -\beta_i^I\right) = \Phi_n(-\boldsymbol{\beta}^I; \boldsymbol{\rho}^I) = [\underline{\Phi}_n, \bar{\Phi}_n] \tag{9}$$

$$P_{Fs}^I \approx P\left(\bigcup_{i=1}^{n} Z_i^I \leqslant 0\right) = 1 - \Phi_n(\boldsymbol{\beta}^I; \boldsymbol{\rho}^I) = 1 - [\underline{\Phi}_n, \bar{\Phi}_n] \tag{10}$$

where $\boldsymbol{\beta}^I = (\beta_1^I, \beta_2^I, \ldots, \beta_n^I)^T$ is an interval column vector associated with the reliability indices of these components. The components of $\boldsymbol{\rho}^I = [\rho_{ij}^I]$ are $\rho_{ij}^I = (\boldsymbol{\alpha}_i^T)^I \boldsymbol{\alpha}_j^I$. Φ_n is the n-dimension standard normal distribution function.

4. Interval model for component structural reliability with uncertain probabilistic characteristics

In this section, the intervals of the structural reliability indices and the probability of failure of the structure's i-th component will be computed.

Define the state function that represents the working state of the i-th component as

$$M_i = g_i(\mathbf{u}) = R_i - S_i \tag{11}$$

where R_i is the resistance, and S_i is the stress resultant caused by the various external loads of the i component structure. Using FORM, the component structural reliability can be calculated as

$$P_{Fi} = P\{M_i < 0\} = P(g_i(\mathbf{u}) < 0) \approx P(\alpha_i u_i < -\beta_i) = \Phi(-\beta_i) \tag{12}$$

in which β_i is structural reliability index

$$\beta_i = \frac{\mu_{Mi}}{\sigma_{Mi}} = \frac{\mu_{ri} - \mu_{si}}{\sqrt{\sigma_{ri}^2 + \sigma_{si}^2}} \tag{13}$$

where μ_{ri} and μ_{si} are the mean values of R_i and S_i, σ_{ri} and σ_{si} are the standard variances of R_i and S_i.

The one-to-one relationship between β_i and P_{Fi} is given by the following equation

$$P_{Fi} = \Phi(-\beta_i) \quad \text{or} \quad \beta_i = -\Phi(P_{Fi}) \tag{14}$$

Let us assume that the mean values and standard variances of R_i and S_i change within the following intervals

$$\underline{\mu}_{ri} \leqslant \mu_{ri} \leqslant \bar{\mu}_{ri}, \quad \underline{\mu}_{si} \leqslant \mu_{si} \leqslant \bar{\mu}_{si}, \quad \underline{\sigma}_{ri} \leqslant \sigma_{ri} \leqslant \bar{\sigma}_{ri}, \quad \underline{\sigma}_{si} \leqslant \sigma_{si} \leqslant \bar{\sigma}_{si} \tag{15}$$

In Eq. (15), $\bar{\mu}_{ri}, \bar{\mu}_{si}$ and $\underline{\mu}_{ri}$, $\underline{\mu}_{si}$ are, respectively, the upper and lower bounds of the mean values μ_{ri} and μ_{si}; $\bar{\sigma}_{ri}, \bar{\sigma}_{si}$ and $\underline{\sigma}_{ri}$, $\underline{\sigma}_{si}$ are, respectively, the upper and lower bounds of the standard variances σ_{ri} and σ_{si}. The probabilistic reliability of the structure with bounded probabilistic characteristics will become a set as follows

$$\Gamma = \left\{ P_{Fi} : \Phi(-\beta_i) = \Phi\left(-\frac{\mu_{ri} - \mu_{si}}{\sqrt{\sigma_{ri}^2 + \sigma_{si}^2}} \right), \begin{array}{l} \underline{\mu}_{ri} \leqslant \mu_{ri} \leqslant \bar{\mu}_{ri}, \underline{\mu}_{si} \leqslant \mu_{si} \leqslant \bar{\mu}_{si} \\ \underline{\sigma}_{ri} \leqslant \sigma_{ri} \leqslant \bar{\sigma}_{ri}, \underline{\sigma}_{si} \leqslant \sigma_{si} \leqslant \bar{\sigma}_{si} \end{array} \right\}. \tag{16}$$

Based on Eq. (14) and the monotonicity of Eq. (12), the extremum problem of the probability of failure of Eq. (16) can be rewritten as the following extremum problem of the structural reliability index

$$\Gamma = \left\{ \beta_i : \beta_i = \frac{\mu_{ri} - \mu_{si}}{\sqrt{\sigma_{ri}^2 + \sigma_{si}^2}}, \begin{array}{l} \underline{\mu}_{ri} \leqslant \mu_{ri} \leqslant \bar{\mu}_{ri}, \underline{\mu}_{si} \leqslant \mu_{si} \leqslant \bar{\mu}_{si} \\ \underline{\sigma}_{ri} \leqslant \sigma_{ri} \leqslant \bar{\sigma}_{ri}, \underline{\sigma}_{s} \leqslant \sigma_{si} \leqslant \bar{\sigma}_{si} \end{array} \right\} \tag{17}$$

We should stress that Γ may be generally complicated geometric shape so it is usually impractical to try to solve the extremum problem of Eq. (17). Instead, in this study, we are interested in the interval containing the structural reliability index with uncertain but bounded probabilistic parameters. Therefore, it is instructive to seek the interval of the reliability index

$$\beta_i^I = [\underline{\beta}_i, \bar{\beta}_i] = [(\beta_i)_{\min}, (\beta_i)_{\max}] \tag{18}$$

where

$$\underline{\beta}_i = (\beta_i)_{\min}, \quad \bar{\beta}_i = (\beta_i)_{\max} \tag{19}$$

Under the condition that μ_{Mi} and σ_{Mi} are not random quantities and hence statistically independent, let us consider the extreme value problem of the structural reliability index β_i. Clearly, the maximum value and the minimum value can be, respectively, expressed as

$$(\beta_i)_{\max} = \frac{(\mu_{Mi})_{\max}}{(\sigma_{Mi})_{\min}}, \quad (\beta_i)_{\min} = \frac{(\mu_{Mi})_{\min}}{(\sigma_{Mi})_{\max}} \tag{20}$$

For the problem at hand we are looking for an

$$\textit{extremum} \quad \mu_{Mi} = \mu_{ri} - \mu_{si} \tag{21}$$

such that

$$\underline{\mu}_{ri} \leqslant \mu_{ri} \leqslant \bar{\mu}_{ri}, \quad \underline{\mu}_{si} \leqslant \mu_{si} \leqslant \bar{\mu}_{si} \tag{22}$$

Since μ_{Mi} is a linear function of variables μ_{ri} and μ_{si}, the maximum value and the minimum value of μ_{Mi} can be easily obtained as

$$(\mu_{Mi})_{\max} = (\mu_{ri})_{\max} - (\mu_{si})_{\min} = \bar{\mu}_{ri} - \underline{\mu}_{si}, \quad (\mu_{Mi})_{\min} = (\mu_{ri})_{\min} - (\mu_{si})_{\max} = \underline{\mu}_{ri} - \bar{\mu}_{si} \tag{23}$$

we are also looking for the solution by finding

$$\textit{extremum} \quad \sigma_{Mi} = \sqrt{\sigma_{ri}^2 + \sigma_{si}^2} \tag{24}$$

with the constraints

$$\underline{\sigma}_{ri} \leqslant \sigma_{ri} \leqslant \bar{\sigma}_{ri}, \quad \underline{\sigma}_{si} \leqslant \sigma_{si} \leqslant \bar{\sigma}_{si} \tag{25}$$

Because σ_{Mi} is a monotonously increasing function of the variables σ_{ri} and σ_{si}, the maximum value and the minimum value of σ_{Mi} can be directly determined as

$$(\sigma_{Mi})_{\max} = \sqrt{(\sigma_{ri})^2_{\max} + (\sigma_{si})^2_{\max}} = \sqrt{(\bar{\sigma}_{ri})^2 + (\bar{\sigma}_{si})^2} \tag{26}$$

$$(\sigma_{Mi})_{\min} = \sqrt{(\sigma_{ri})^2_{\min} + (\sigma_{si})^2_{\min}} = \sqrt{(\underline{\sigma}_{ri})^2 + (\underline{\sigma}_{si})^2} \tag{27}$$

Thus, in terms of Eqs. (23), (26), (27), (20) can be rewritten as

$$(\beta_i)_{\max} = \frac{(\mu_{Mi})_{\max}}{(\sigma_{Mi})_{\min}} = \frac{(\mu_{ri})_{\max} - (\mu_{si})_{\min}}{\sqrt{(\sigma_{ri})^2_{\min} + (\sigma_{si})^2_{\min}}} = \frac{\bar{\mu}_{ri} - \underline{\mu}_{si}}{\sqrt{(\underline{\sigma}_{ri})^2 + (\underline{\sigma}_{si})^2}} \tag{28}$$

$$(\beta_i)_{\min} = \frac{(\mu_{Mi})_{\min}}{(\sigma_{Mi})_{\max}} = \frac{(\mu_{ri})_{\min} - (\mu_{si})_{\max}}{\sqrt{(\sigma_{ri})^2_{\max} + (\sigma_{si})^2_{\max}}} = \frac{\underline{\mu}_{ri} - \bar{\mu}_{si}}{\sqrt{(\bar{\sigma}_{ri})^2 + (\bar{\sigma}_{si})^2}} \tag{29}$$

Consequently, according to the Eq. (14), the best possible value (or the upper bound) and the worst possible value (or the lower bound) of the structural reliability can be calculated, respectively, as

$$\underline{P}_{Fi} = \Phi(-\bar{\beta}_i) \tag{30}$$

and

$$\overline{P}_{Fi} = \Phi(-\underline{\beta}_i) \tag{31}$$

5. Numerical examples

Example 1. Consider a 14-bar 2D truss structure as shown in Fig. 1. The elastic moduli and cross-sectional areas for all members are the same, which are 70 GPa and 0.004 m^2, respectively. The data of loads and resistances of the components are listed in Table 1. The mean value and the standard deviation of the quantities R_i and S_i are uncertain and changing within the following intervals, respectively,

$$\mu_{ri} = [\mu_{ri}(1-\alpha_1), \mu_{ri}(1+\alpha_1)], \quad \mu_{si} = [\mu_{si}(1-\alpha_2), \mu_{si}(1+\alpha_2)] \tag{32}$$

$$\sigma_{ri} = [\sigma_{ri}(1-\alpha_3), \sigma_{ri}(1+\alpha_3)], \quad \sigma_{si} = [\sigma_{si}(1-\alpha_4), \sigma_{si}(1+\alpha_4)] \tag{33}$$

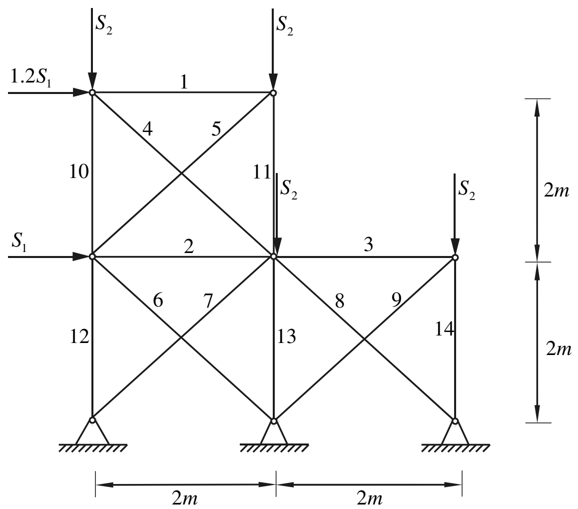

Fig. 1. 14-bar 2D truss structure.

Table 1
The statistic data of loads and resistance of the 14-bar 2D truss structure

	R_1 (MPa)	R_2 (MPa)	R_3 (MPa)	R_4 (MPa)	R_5 (MPa)	S_1 (KN)	S_2 (KN)
Mean value	±288	±216	±180	±360	±540	125	20
Coefficient of variance	0.15	0.15	0.15	0.15	0.15	0.2	0.2
Components	1, 2, 3	4, 5	6, 8	7, 9, 10, 11	12, 13, 14	–	–

where α_1, α_2, α_3 and α_4 are coefficients in the range of α, which is 0–0.1. We will limit ourselves by discussing the case $\alpha_i = \alpha(i = 1, 2, 3, 4)$ in this study. Assume that the stiffness and the loads have normal distributions, and all components are independent. The buckling effects of the compressed components are neglected. The failure probability interval of the system is our goal of calculation.

According to the Branch-Bound Method (BBM), the 14 main failure modes of the system can be obtained. The search tree of this method is shown in Fig. 2, which gives the failure paths of main failure modes. For example, ④ → ①describes one of the failure paths, and ④and ①, respectively, denotes the numbering of the failure components. The lower bounds and the upper bounds of the reliability index and failure probability corresponding to the 14 main failure modes are shown in Table 2. For this reason, the nominal value of the component failure probability is used to search the main failure modes in the BBM. The failure probabilities of the failure modes which are changing with the parameter α are shown from Figs. 3–5. From Figs. 3–5, we can see that the failure probability intervals of the main failure modes are all increasing as the parameter α increases, as we expected.

Example 2. Consider a 32-bar 3D truss structure as shown in Fig. 6. The elastic modulus of bars is 70 GPa. The cross-sectional areas of bars are, respectively, 0.15 m^2 for 1st–16th bars and 0.08 m^2 for 17th–32th bars. The data of resistances and loads of the components are listed in Table 3. The uncertain parameters are the same with Example 1 and the same range of the uncertain coefficients α. Using the BBM, we get the 15 failure modes of the system. The search tree is shown in Fig. 7. The lower bounds and upper bounds of the reliability indices and the failure probability are listed in the Table 4. The failure probabilities of the failure modes which vary with the parameter α are shown from Figs. 8–10. The result of Example 2 is similar to Example 1. The two examples have shown the feasibility of the proposed probability interval reliability of structure system.

6. Conclusions

The typical reliability theory is based on the statistic theory. Because of the requirement of the probability theory, the probability reliability method needs a large amount of statistic data. On the other hand, we know that we can get or estimate the approximate ranges for some parameters with little errors. For these reasons,

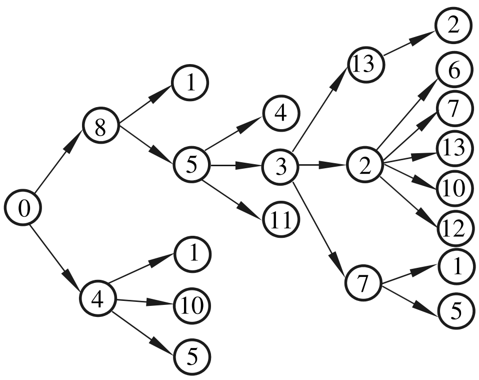

Fig. 2. Search tree of the main failure modes for the 14-bar 2D truss structure.

Table 2
The lower and upper bounds of the reliability index and the failure probability of the main failure modes the 14-bar 2D truss structure ($\alpha = 0.1$)

Sequence number of the failure mode	Failure path	$\underline{\beta}_i$	$\overline{\beta}_i$	$\underline{P}_{fl}$	$\overline{P}_{fl}$
1	8–1	3.896	6.489	4.890×10^{-5}	4.324×10^{-11}
2	8–5–11	3.896	6.489	4.890×10^{-5}	4.324×10^{-11}
3	8–5–3–2–6	3.896	6.489	4.890×10^{-5}	4.324×10^{-11}
4	8–5–3–2–7	3.896	6.489	4.890×10^{-5}	4.324×10^{-11}
5	8–5–3–2–13	4.135	6.755	1.771×10^{-5}	7.163×10^{-12}
6	8–5–3–2–10	4.304	6.940	8.387×10^{-6}	1.967×10^{-12}
7	8–5–3–2–12	4.541	7.197	2.799×10^{-6}	3.086×10^{-13}
8	8–5–3–2–4	4.570	7.228	2.438×10^{-6}	2.453×10^{-13}
9	8–5–3–7–2	3.950	6.548	3.920×10^{-5}	2.912×10^{-11}
10	8–5–3–7–13	4.524	7.179	3.031×10^{-6}	3.523×10^{-13}
11	8–5–3–13–2	4.6322	7.295	1.809×10^{-6}	1.498×10^{-13}
12	8–5–4	4.073	6.686	2.320×10^{-6}	1.150×10^{-11}
13	4–5	4.339	6.978	7.159×10^{-6}	1.151×10^{-12}
14	4–1	4.339	6.978	7.159×10^{-6}	1.151×10^{-12}
15	4–10	4.532	7.187	2.922×10^{-6}	3.314×10^{-13}

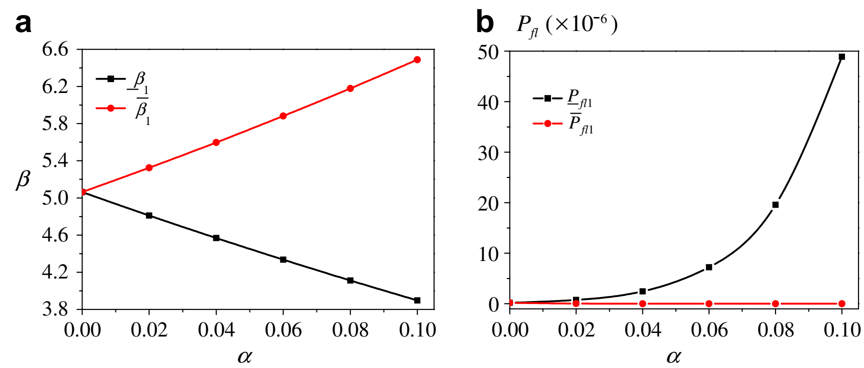

Fig. 3. The reliability index (a) and the failure probability; (b) intervals of the 14-bar 2D truss structure of the main failure modes (1) increasing with α.

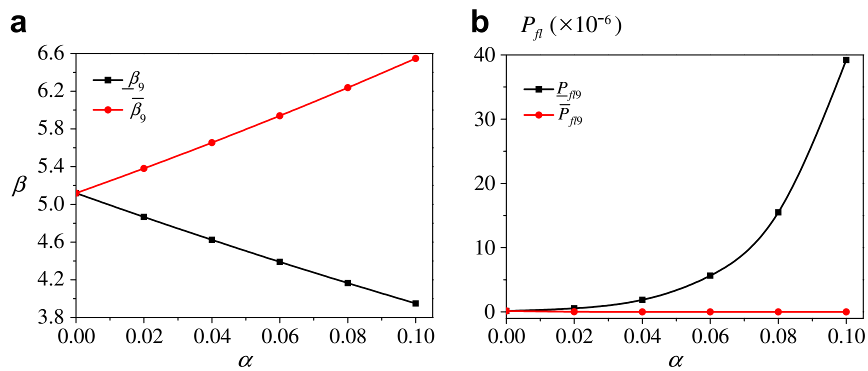

Fig. 4. The reliability index (a) and the failure probability; (b) intervals of the 14-bar 2D truss structure of the main failure mode (9) increasing with α.

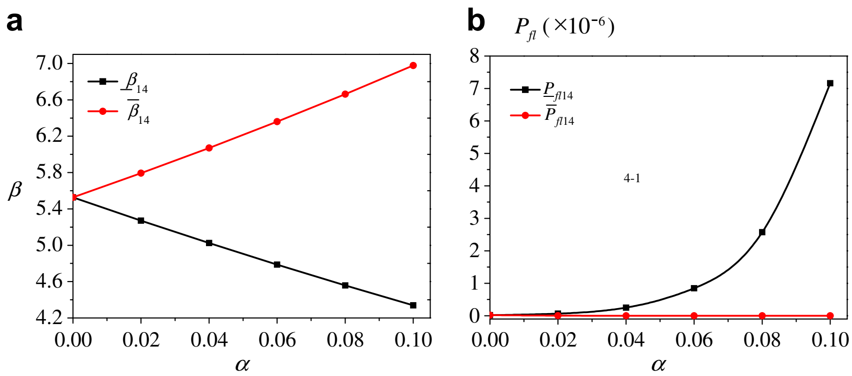

Fig. 5. The reliability index (a) and the failure probability; (b) intervals of the 14-bar 2D truss structure of the main failure mode (14) increasing with α.

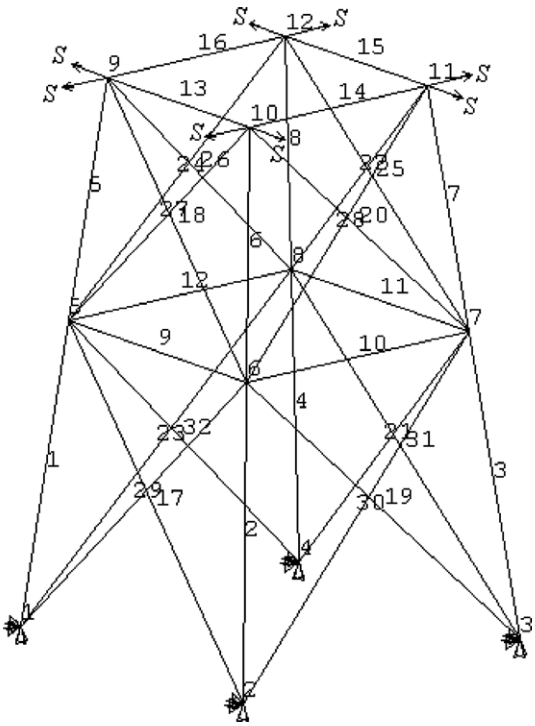

Fig. 6. 32-bar 3D truss structure.

Table 3
The statistic data of resistance and loads of the 32-bar 3D truss structure

	Resistance R (MPa)	Load S (MN)
Mean value	±325.0	30.0
Coefficient of variance	0.15	0.15
Components	① ~ ㉜	

researchers want to develop a new reliability analysis approach which can avoid the shortage of probability method and make use of the approximate ranges of parameters.

The probability interval reliability presented in this article is based on this idea. It avoids the requirement of a large mount of data and inherits the developed theory and method of the classical probability

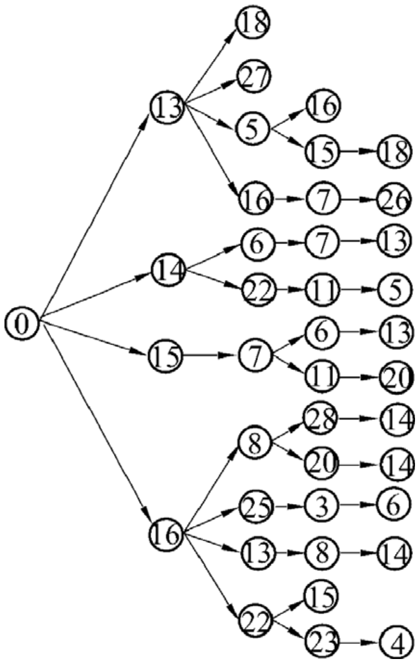

Fig. 7. Search tree of the main failure modes for the 32-bar 3D truss structure.

Table 4
The lower and upper bounds of the reliability indices and the failure probability of the main failure modes for the 32-bar 3D truss structure ($\alpha = 0.1$)

Sequence number of the failure mode	Failure path	$\underline{\beta}_i$	$\overline{\beta}_i$	$\underline{P}_{fl}$	$\overline{P}_{fl}$
1	13–18	1.4056	3.7407	9.176×10^{-5}	7.993×10^{-2}
2	13–27	1.4056	3.7407	9.176×10^{-5}	7.993×10^{-2}
3	13–5–16	1.4056	3.7407	9.176×10^{-5}	7.993×10^{-2}
4	13–5–15–18	1.4056	3.7407	9.176×10^{-5}	7.993×10^{-2}
5	14–6–7–13	1.4088	3.7444	9.041×10^{-5}	7.945×10^{-2}
6	15–7–6–13	1.4088	3.7444	9.041×10^{-5}	7.945×10^{-2}
7	16–8–28–14	1.4128	3.7491	8.875×10^{-5}	7.886×10^{-2}
8	16–25–3–6	1.4271	3.7657	8.306×10^{-5}	7.677×10^{-2}
9	13–16–7–26	1.4471	3.7887	7.571×10^{-5}	7.394×10^{-2}
10	15–7–11–20	1.7329	4.1182	1.909×10^{-5}	4.155×10^{-2}
11	16–8–20–14	2.0022	4.4263	4.792×10^{-6}	2.263×10^{-2}
12	16–13–8–14	2.1021	4.5401	2.812×10^{-6}	1.777×10^{-2}
13	16–22–15	2.2003	4.6516	1.647×10^{-6}	1.389×10^{-2}
14	16–22–23–4	2.3123	4.7785	8.833×10^{-7}	1.038×10^{-2}
15	14–22–11–5	2.5916	5.0929	1.763×10^{-7}	4.777×10^{-3}

approach. The uncertain parameters are described as interval variables in this approach. In terms of the theory and method of the interval mathematics, the expressions of reliability index, series system failure probability, parallel system failure probability are obtained in the form of intervals. The method developed in this paper is the combination of probability reliability theory and interval analysis theory. It can solve the reliability problem when there is no sufficient data to perform the probability reliability approach. In the examples of this paper, we can see that the result of the interval reliability is perfect when the intervals of the parameters are in the appropriate ranges. From the two examples, we can see the feasibility of this new method.

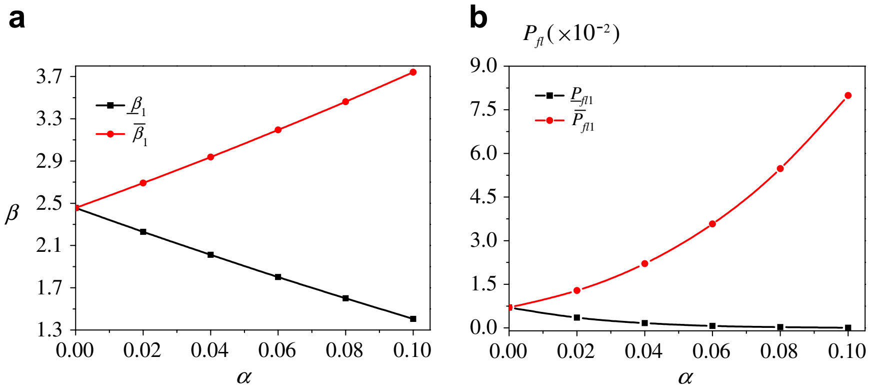

Fig. 8. The reliability index (a) and the failure probability; (b) intervals of the 32-bar 3D truss structure of the main failure modes (1) increasing with α.

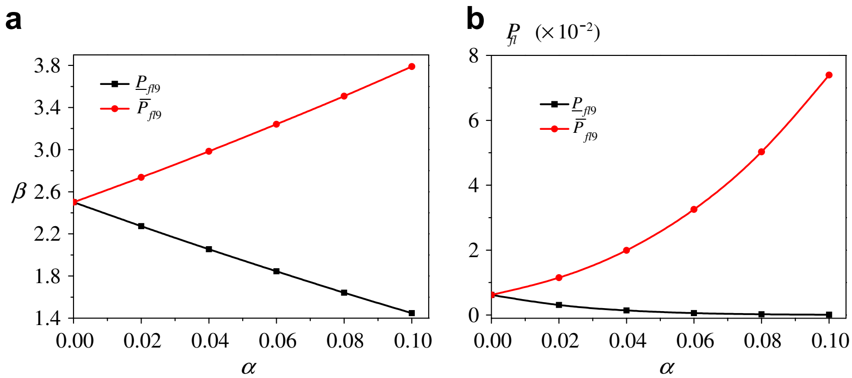

Fig. 9. The reliability index (a) and the failure probability; (b) intervals of the 32-bar 3D truss structure of the main failure modes (9) increasing with α.

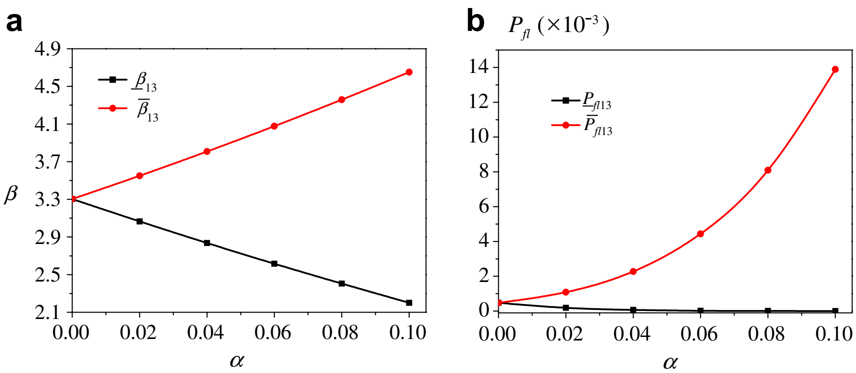

Fig. 10. The reliability index (a) and the failure probability; (b) intervals of the 32-bar 3D truss structure of the main failure modes (13) increasing with α.

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