Post-print of: Su, L. et al. "Modeling and applications of metamaterial transmission lines loaded with pairs of coupled complementary split ring resonators (CSRRs) " in IEEE antennas and wireless propagation letters, vol. 15 (2016), p. 154-157. DOI 10.1109/LAWP.2015.2435656

Modeling and Applications of Metamaterial Transmission Lines loaded with Pairs of Coupled Complementary Split Ring Resonators (CSRRs)

Lijuan Su, Jordi Naqui, *Member, IEEE*, Javier Mata-Contreras, Ferran Martín, *Fellow, IEEE*

Abstract **This paper is focused on the modeling, analysis and applications of microstrip lines loaded with pairs of electrically coupled complementary split ring resonators (CSRRs). Typically, these epsilon negative (ENG) metamaterial transmission lines are implemented by loading the line with a single CSRR (etched beneath the conductor strip) in the unit cell. This provides a stop band in the vicinity of the CSRR resonance. However, by loading the line with a pair of CSRRs per unit cell, it is possible to either implement a dual-band ENG transmission line (useful, for instance, as a dual-band notch filter), provided the CSRRs are tuned at different frequencies, or to design microwave sensors and comparators based on symmetry disruption (in this case by using identical CSRRs and by truncating symmetry by different means, e.g., asymmetric dielectric loading). The design of these CSRR-based structures requires an accurate circuit model able to describe the line, the resonators and the different coupling mechanisms (i.e., line-to-resonator and inter-resonator coupling). Thus, a lumped element equivalent circuit is proposed and analyzed in detail. The model is validated by comparison to electromagnetic simulations and measurements. A proof-of-concept of a differential sensor for dielectric characterization is proposed. Finally, the similarities of these structures with coplanar waveguide transmission lines loaded with pairs of SRRs are pointed out.**

***Index Terms*– Electromagnetic metamaterials, metamaterial transmission lines, complementary split ring resonators (CSRRs).**

I. Introduction

Metamaterial transmission lines based on complementary split ring resonators (CSRRs), first proposed in [1],[2], have found many applications in RF/microwave engineering, including filters [3]-[6], multiband components [7], enhanced bandwidth components [8], leaky wave antennas [9], etc. Typically, these lines are implemented by loading a microstrip line with the CSRRs etched in the ground plane, beneath the conductor strip, and with their symmetry plane orthogonal to the line axis. Under these conditions, line-to-resonator coupling is purely electric, and the lumped element circuit model (including only electric coupling) was first proposed in [10]. The effects of CSRR rotation (i.e., mixed electric and magnetic coupling) and coupling between CSRRs of adjacent cells (bandwidth enhancement), not included in the former model, were studied in [11] and [12], respectively.

In these previous studies and in the reported applications of CSRR-loaded lines, only one CSRR in the unit cell was considered. However, in microstrip lines loaded with two CSRRs per unit cell interesting possibilities arise. Specifically, by loading the line with different CSRRs, a dual-band epsilon negative (ENG) metamaterial transmission line results. These lines can be useful, for instance, as dual-band bandstop or notched filters. Conversely, by loading the line with identical CSRR in the unit cell, and by considering a single unit cell in the structure, the resulting (single) transmission zero splits into two notches if symmetry is truncated, e.g., by an asymmetric dielectric load. Thus, these symmetric CSRR-pair-loaded lines have potential applications as differential sensors or comparators.

Necessarily, the distance between the CSRRs forming the pair must be small (otherwise the coupling between the line and the CSRRs would be negligible). Hence, the circuit model of these lines should account not only for the line, CSRRs, and coupling between the line and the resonators (as usual [10]), but also for inter-resonator coupling. In this paper, the lumped element equivalent circuit model of these microstrip lines loaded with pairs of CSRRs is reported and analyzed in detail, and a proof-of-concept of a comparator to detect differences between a sample and a reference is proposed. In Section II, the model is presented and analyzed, and it is compared to the model of coplanar waveguides (CPWs) loaded with pairs of SRRs [13]. Model validation through parameter extraction is carried out in Section III, whereas the use of a symmetric structure as a comparator is demonstrated in Section IV. Finally, the main conclusions are highlighted in Section V.

II. Circuit Model and Analysis

The topology and the proposed lumped element equivalent circuit model of the considered structures (unit cell) are depicted in Fig. 1(a) and (b), respectively. The line is modeled by the inductance L and the capacitance C. C_{c1} and C_{c2} account for the electric coupling between the line and the CSRRs, described by the resonant tanks L_1-C_1 and L_2-C_2, and C_M accounts for their mutual electric coupling. The model, valid as long as the CSRRs are electrically small, considers the general case of a microstrip line loaded with different CSRRs (asymmetric structure). Besides the fact that there are two coupled CSRRs per unit cell, there is a significant

This work has been supported by MINECO (Spain) under project TEC2013-40600-R, by AGAUR-Generalitat de Catalunya under project 2014SGR-157, and by FEDER funds. Ferran Martín is in debt to ICREA for supporting his work. Lijuan Su thanks CSC (China scholarship council) Grant for supporting her work in UAB.

The authors are with GEMMA/CIMITEC, Departament d'Enginyeria Electrònica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain (e-mail: Ferran.Martin@uab.es).

difference between the model of Fig. 1(b) and the model of the microstrip line loaded with a single CSRR [10]. In [10], the capacitance of the line was considered to be the coupling capacitance between the line and the CSRR. The reason is that if the single CSRR is etched in the ground plane with its center aligned with the line axis, the conductor strip is located above the inner metallic region of the CSRR, and the electric field lines generated by the line entirely penetrate (at least to a first order approximation) the inner metallic region of the CSRRs (this is valid provided the conductor strip -unit cell- does not extend significantly beyond the extremes of the CSRR). However, in the structure of Fig. 1(a) not all the electric field lines penetrate the inner metallic regions of the CSRRs, hence being necessary to include a capacitor, C, between the conductor strip and ground. Strictly speaking, this is not the line capacitance, but the capacitance between the inter-resonator metallic region and the conductor strip (note that this capacitance should increase by increasing the distance between CSRRs, at the expense of a decrease in C_{c1} and C_{c2}).

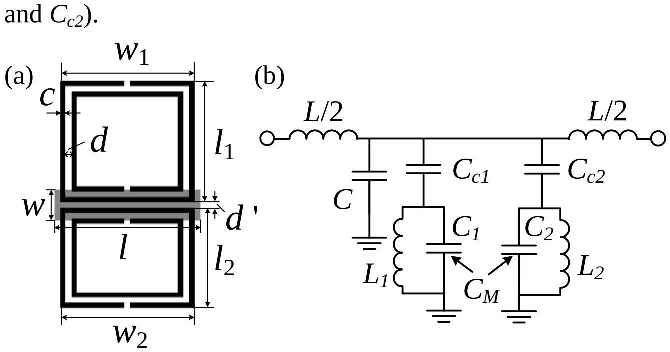

Fig. 1. Typical topology of a microstrip line loaded with a pair of CSRRs (a) and lumped element circuit model (b). The CSRRs (etched in the ground plane) are depicted in black and the conductor strip (upper metal) is depicted in grey.

In order to find the transmission zeros in the circuit of Fig. 1(b), the reactance of the shunt branch is forced to be zero (note that C has not any influence on this calculation). This leads us (by using the equivalence shown in Fig. 2 [14] and the Δ-T transform, and after some tedious calculations) to the following bi-quadratic equation:

$$A\omega^4 + B\omega^2 + D = 0 \tag{1}$$

where ω is the angular frequency and

$$A = L_1L_2\{C_M(C_{c1}+C_{c2})(C_{c1}+C_1)(C_{c2}+C_2) - C_{c1}C_{c2}C_M^2\} +$$
$$+ L_1L_2\{C_{c1}C_{c2}(C_{c1}+C_1)(C_{c2}+C_2) - (C_{c1}+C_{c2})C_M^3\} =$$
$$= D\cdot L_1L_2\{(C_{c1}+C_1)(C_{c2}+C_2) - C_M^2\}$$
$$B = -D\cdot\{L_1(C_{c1}+C_1) + L_2(C_{c2}+C_2)\}$$
$$D = C_{c1}C_M + C_{c2}C_M + C_{c1}C_{c2}$$

Since $D > 0$, the solution of (1) is equivalent to the solution of

$$L_1L_2\{(C_{c1}+C_1)(C_{c2}+C_2) - C_M^2\}\omega^4 -$$
$$-\{L_1(C_{c1}+C_1) + L_2(C_{c2}+C_2)\}\omega^2 + 1 = 0 \tag{2}$$

namely,

$$\omega_\pm^2 = \frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2-\omega_2^2)^2 + 4C_M^2\omega_1^4\omega_2^4L_1L_2}}{2[1 - C_M^2\omega_1^2\omega_2^2L_1L_2]} \tag{3}$$

where

$$\omega_i = \frac{1}{\sqrt{L_i(C_i + C_{ci})}} \tag{4}$$

with i = 1,2, are the transmission zero frequencies of the isolated resonators (i.e., without inter-resonator coupling).

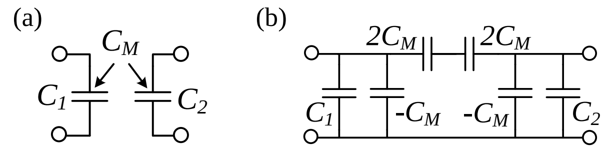

Fig. 2. Two-port with mutual capacitive coupling (a) and its equivalent π-circuit (b).

Note that if inter-resonator coupling is zero (C_M = 0), the two solutions of (3) are simply ω_1 and ω_2, an expected result since at these frequencies one of the two (uncoupled) parallel branches of the circuit of Fig. 1(b) is shorted to ground. Let us now discuss three specific situations considering that $C_M \neq 0$.

A. Symmetric case

If the structure is symmetric with regard to the line axis, then $C_{c1} = C_{c2} = C_c$, $L_1 = L_2 = L_r$, and $C_1 = C_2 = C_r$ (giving also $\omega_1 = \omega_2 = \omega_0$), and the two solutions of (3) are:

$$\omega_\pm = \frac{\omega_o}{\sqrt{1 \mp \dfrac{C_M}{C_r + C_c}}} \tag{5}$$

However, ω_- is not actually a physical solution, since the shunt branch reactance presents a pole at this frequency, resulting in a finite reactance. Indeed, the single transmission zero frequency (ω_+) that arises when the CSRR-loaded line is symmetric can be easily inferred by direct inspection of the circuit of Fig. 1(b) and the equivalence of Fig. 2. Note that the coupling between CSRRs, for this symmetric case, has the effect of increasing the transmission zero frequency.

B. Asymmetric case (different CSRRs with same resonance frequency)

Let us now consider an asymmetric structure (i.e., with different coupled CSRRs, or $L_1 \neq L_2$, $C_1 \neq C_2$), but satisfying the condition $\omega_1 = \omega_2 = \omega_0$. In this case, expression (3) gives:

$$\omega_\pm = \frac{\omega_o}{\sqrt{1 \mp \dfrac{C_M}{\sqrt{(C_1 + C_{c1})(C_2 + C_{c2})}}}} \tag{6}$$

namely, an expression formally identical to (5). In general the two mathematical solutions given by (6) are both physical solutions (none of them correspond to poles of the shunt reactance), and the corresponding transmission zeros are located to the left (ω_-) and right (ω_+) of ω_0. However, if the following balance condition is satisfied

$$\frac{C_{c1}}{C_{c2}} = \sqrt{\frac{C_1 + C_{c1}}{C_2 + C_{c2}}} = \sqrt{\frac{L_2}{L_1}} \tag{7}$$

then only one transmission zero (at ω_+) is expected, since the reactance exhibits a pole at ω_-.

C. General case

For the general case, i.e., an asymmetric structure with arbitrary resonator frequencies ($L_1 \neq L_2$, $C_1 \neq C_2$) and $\omega_1 \neq \omega_2$, the transmission zeros are given by the two solutions of (3). The coupling capacitance C_M enhances the distance between the transmission zeros, i.e.,

$$\omega_+^2 - \omega_-^2 = \frac{\sqrt{(\omega_1^2 - \omega_2^2)^2 + 4C_M^2 \omega_1^4 \omega_2^4 L_1 L_2}}{[1 - C_M^2 \omega_1^2 \omega_2^2 L_1 L_2]} > \omega_1^2 - \omega_2^2 \quad (8)$$

Moreover, $\omega_+ > \omega_1$ and $\omega_- < \omega_2$, where it is assumed that $\omega_1 > \omega_2$.

D. Comparison to the model of SRR-loaded CPW transmission lines

Despite the fact that CPW structures loaded with pairs of magnetically coupled split ring resonators (SRRs) are described by a circuit model completely different from the one of Fig. 1(b) [13], the behavior of these SRR- and CSRR-loaded lines, inferred from the circuit models, is very similar (although not identical). Indeed, the general solution (3), as well as the mathematical solutions corresponding to the considered particular cases (expressions 5, 6 and condition 7) are identical to the equivalent solutions in SRR-loaded lines, provided the following mapping holds:

$$C_M \leftrightarrow M' \quad (9a)$$

$$L_{1,2} \leftrightarrow C_{1,2} \quad (9b)$$

$$C_{1,2} + C_{c1,2} \leftrightarrow L_{1,2} \quad (9c)$$

where the left and right hand sides refer to the elements of the circuit model of the CSRR-loaded microstrip line [Fig. 1(b)] and SRR-loaded CPW [13], respectively.

One difference between both models concerns the single transmission zero for the symmetric case or for the asymmetric case subjected to the balance condition. As mentioned above, for CSRR-loaded lines the physical solution is the upper frequency, whereas it is the lower frequency for SRR-loaded lines. Thus, for microstrip lines loaded with pairs of identical and symmetrically etched CSRRs, the transmission zero appears to the right of ω_0, whereas it appears to the left of ω_0 in symmetric SRR-loaded CPW transmission lines. Note, however that ω_0, ω_1 and ω_2 are defined by (4) in this work, whereas in [13], these angular frequencies are the intrinsic resonator (SRR) frequencies.

It is also worth to mention that, whereas in SRR-loaded CPWs the position of the transmission zeros does not depend on the mutual coupling between the line and the resonators, this does not hold in microstrip lines loaded with pairs of CSRRs, where the coupling capacitors C_{c1} and C_{c2} (or C_c for the symmetric case) have influence on the location of the transmission zeros (through ω_1 and ω_2).

III. Validation

To validate the model we have considered three different structures. Two of them are symmetric, the unique difference being the lengths $l_1 = l_2$ [see Fig. 1(a)],which are set to $l_1 = l_2 =$ 3.8 mm in one case (A), and to $l_1 = l_2 = 4.6$ mm in the other case (B). The asymmetric structure (case C) is implemented by considering the two previous CSRRs (i.e., $l_1 = 4.6$ mm and $l_2 = 3.8$ mm). The other parameters, in reference to Fig. 1(a), are $W = 1.18$ mm, $l = 5.2$ mm, $c = d = 0.2$ mm, $w_1 = w_2 = 4.8$ mm, and $d' = 0.2$ mm. The considered substrate is *Rogers RO3010* with thickness $h = 1.27$ mm and dielectric constant ε_r =10.2. The simulated (without losses) frequency responses of these structures, inferred from the *Keysight Momentum* commercial software, are depicted in Fig. 3. Parameter extraction has been done following the procedure explained in the next paragraph.

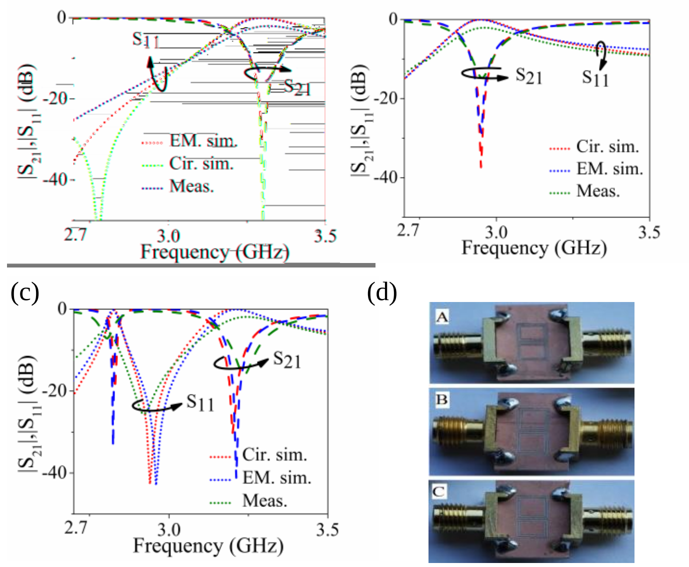

Fig. 3. Electromagnetic simulation, circuit simulation and measured responses of the three considered CSRR-loaded microstrip structures. (a) symmetric case A; (b) symmetric case B; (c) asymmetric case C; (d) photographs of symmetric case A, symmetric case B and asymmetric case C.

For the symmetric structures, the equivalent circuit model can be reduced to the one depicted in Fig. 4, and parameter extraction is realized following the procedure first published in [15]. Note, however, that the fact that there is an additional reactive element in the model (the capacitor C) forces us to include an additional condition for parameter extraction. However, since there are two frequencies where the phase of S_{21} is -90^o, the condition $Z_s(\omega) = -Z_p(\omega)$ is used twice, and the five parameters of the model can be determined, i.e., L, C, C_c, L_r and $C_r - C_M$. Following this procedure, it is not possible to independently infer C_r and C_M. However, by repeating this procedure for the other symmetric structure, we can obtain the corresponding set of parameters. Finally, C_M is used as fitting parameter in the asymmetric structure, to adjust the position of the transmission zeros, and once C_M is known, C_r for each symmetric structure (or C_1 and C_2 in the asymmetric one) can be inferred. The extracted parameters for the three structures are shown in table I.

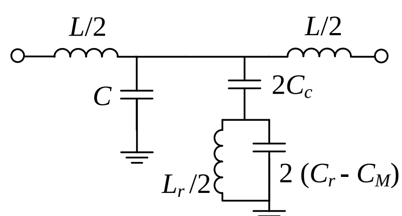

Fig. 4. Lumped element circuit model for the symmetric CSRR-loaded line.

TABLE I. EXTRACTED CIRCUIT PARAMETERS

	L (nH)	*C* (pF)	C_1-C_2 (pF)	L_1-L_2 (nH)	$C_{c1,2}$ (pF)	C_M (pF)
A	3.47	0.086	1.96	1.10	0.336	0.177
B	3.53	0.092	2.27	1.18	0.360	0.177
C	3.50	0.085	1.96-2.27	1.10-1.18	0.347	0.177

The circuit simulations for the three considered cases are also depicted in Fig. 3, where it can be appreciated the good agreement with the lossless electromagnetic simulations. The three structures have been fabricated and the measured responses (inferred from the *Agilent Technologies N5221A* vector network analyzer) are also depicted in Fig. 3. The slight discrepancies with the circuit and electromagnetic simulations are due to fabrication related tolerances and to the fact that losses have been excluded in the simulations. Nevertheless, with the results of Fig. 3, it is clear that the proposed model [Fig. 1(b)] is validated.

We would like to mention that in SRR-loaded CPWs, the increase in frequency splitting (asymmetric structures) caused by shortening the distance between SRRs (hence enhancing their coupling) was confirmed through electromagnetic simulation and experiment. In microstrip lines loaded with pairs of CSRRs, this effect has also been corroborated, but by varying the inter-CSRR space *d*', not only C_M experiences changes (as expected), but also C_{c1}, C_{c2}, C, L_1, L_2, C_1 and C_2 are modified. This means that when *d'* is reduced, the effects (increase of frequency splitting) cannot be merely attributed to inter-CSRR coupling enhancement.

IV. PROOF OF CONCEPT OF A COMPARATOR

According to Fig. 3(c), it is clear that the asymmetric structures can be used as dual-band notched filters. However, an alternative interesting application concerns differential sensors and comparators on the basis of disruption of symmetry. Namely, if a symmetric CSRR-loaded microstrip line is loaded with an asymmetric load, this asymmetry will cause a frequency splitting that can be easily detected. Thus, it is possible, for instance, to compare dielectric loads to a reference, in order to detect defects, abnormalities, etc. As a proof of concept, we report here the frequency response of the symmetric structure (case A), where one of the CSRR has been loaded with a dielectric slab of permittivity identical to that of the substrate (10.2), thickness 1.27 mm and dimensions 7.5 mm × 6.6 mm. As can be appreciated, two notches, indicative of the asymmetric loading of the structure, appear.

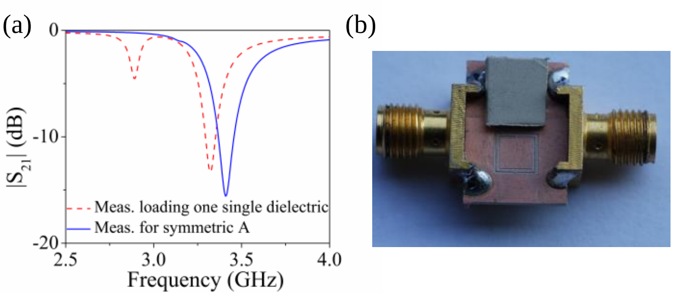

Fig. 5. (a) Measured frequency response of the symmetric CSRR-loaded microstrip structure corresponding to case A, loaded with a dielectric slab, placed on top of one of the CSRRs; (b) photograph of the symmetric case A loaded with a dielectric slab in the upper CSRR.

V. CONCLUSION

We have proposed and validated a circuit model for microstrip lines loaded with pairs of electrically coupled CSRRs. The model, valid for both symmetric and asymmetric structures, has been analyzed in detail, and the similarities with CPW transmission lines loaded with pairs of magnetically coupled SRRs have been pointed out. The model has been validated by parameter extraction and comparison to electromagnetic simulations and experimental data, by considering two symmetric and one asymmetric structures. Finally, a proof of concept of a comparator has been reported.

REFERENCES

[1] F. Falcone, T. Lopetegi, J.D. Baena, R. Marqués, F. Martín and M. Sorolla, "Effective negative-ε stop-band microstrip lines based on complementary split ring resonators", *IEEE Microw. Wirel. Comp. Lett*., vol. 14, pp. 280-282, Jun. 2004.

[2] F. Falcone, T. Lopetegi, M.A.G. Laso, J.D. Baena, J. Bonache, R. Marqués, F. Martín, M. Sorolla, "Babinet principle applied to the design of metasurfaces and metamaterials", *Phys. Rev. Lett*., vol. 93, paper 197401, Nov. 2004.

[3] J. Bonache, F. Martín, I. Gil, J. García-García, R. Marqués and M. Sorolla, "Microstrip bandpass filters with wide bandwidth and compact dimensions", *Microw. Opt. Technol. Lett*., vol. 46, pp. 343-346, Aug. 2005.

[4] J. Bonache, F. Martín, J. García-García, I. Gil, R. Marqués and M. Sorolla, "Ultra wide band pass filtres (UWBPF) based on complementary split rings resonators", *Microw. Opt. Technol. Lett*., vol. 46, pp.283-286, Aug. 2005.

[5] P. Mondal, M.K. Mandal, A. Chaktabarty, S. Sanyal, "Compact bandpass filters with wide controllable fractional bandwidth", *IEEE Microw. Wireless Compon. Lett.*, vol. 16, pp, 540–542, Oct. 2006.

[6] J. Bonache, I. Gil, J. García-García, F. Martín, "Novel Microstrip Band Pass Filters Based on Complementary Split Rings Resonators", *IEEE Trans. Microw. Theory Techn*., vol. 54, pp. 265-271, Jan. 2006.

[7] J. Bonache, G. Sisó, M. Gil, A. Iniesta, J. García-Rincón and F. Martín, "Application of composite right/left handed (CRLH) transmission lines based on complementary split ring resonators (CSRRs) to the design of dual band microwave components", *IEEE Microw. Wirel. Compon. Lett*., vol. 18, pp. 524-526, Aug. 2008.

[8] G. Sisó, J. Bonache, M. Gil and F. Martín, "Application of resonant-type metamaterial transmission lines to the design of enhanced bandwidth components with compact dimensions", *Microw. Opt. Technol. Lett.*, vol. 50, pp. 127-134, Jan. 2008.

[9] S. Eggermont, I. Huynen, "Leaky wave radiation phenomena in metamaterial transmission lines based on complementary split ring resonators," *Microw. Opt. Techn. Lett.*, vol. 53, pp. 2025-2029, Sep. 2011.

[10] J.D. Baena, J. Bonache, F. Martín, R. Marqués, F. Falcone, T. Lopetegi, M.A.G. Laso, J. García, I Gil, M. Flores-Portillo and M. Sorolla, "Equivalent circuit models for split ring resonators and complementary split rings resonators coupled to planar transmission lines", *IEEE Trans. Microw. Theory Techn.*, vol. 53, pp. 1451-1461, Apr. 2005.

[11] J. Naqui, M. Durán-Sindreu, and F. Martín, "Modeling split ring resonator (SRR) and complementary split ring resonator (CSRR) loaded transmission lines exhibiting cross polarization effects", *IEEE Ant. Wirel. Propag. Lett.*, vol. 12, pp. 178-181, 2013.

[12] J. Naqui, M. Durán-Sindreu, A. Fernández-Prieto, F. Mesa, F. Medina, and F. Martín, "Multimode propagation and complex waves in CSRR-based transmission line metamaterials", *IEEE Ant. Wirel. Propag. Lett.*, vol. 11, pp. 1024-1027, 2012.

[13] L. Su, J. Naqui, J. Mata-Contreras and F. Martín "Modeling metamaterial transmission lines loaded with pairs of coupled split ring resonators", *IEEE Ant. Wirel. Propag. Lett*., vol. 14, pp. 68-71, 2015.

[14] J-S. Hong and M.J. Lancaster, *Microstrip filters for RF/microwave applications*, John Wiley, New Jersey, 2001.

[15] J. Bonache, M. Gil, I. Gil, J. Garcia-García and F. Martín, "On the electrical characteristics of complementary metamaterial resonators", *IEEE Microw. Wirel. Compon. Lett*., vol. 16, pp. 543.545, Oct. 2006.