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Carmichael's lambda function

by

PAUL ERDŐS (Budapest), CARL POMERANCE* (Athens, Ga.) and
ERIC SCHMUTZ* (Philadelphia, Penn.)

1. Introduction. Let $\lambda(n)$ be the universal exponent for the group of residues mod n that are coprime to n. A more explicit definition of λ is:

$$\begin{aligned} &\lambda(p^e) = \phi(p^e) = p^{e-1}(p-1) && \text{if } p \text{ is an odd prime,}\\ &\lambda(2^e) = \phi(2^e) && \text{if } e = 0, 1, \text{ or } 2,\\ &\lambda(2^e) = \tfrac{1}{2}\phi(2^e) && \text{if } e \geqslant 3 \end{aligned}$$

and finally,

By taking a subsequence $\langle n_{i_j} \rangle_j$, we can obtain a sequence that is increasing and

From (15) we get

$$\sum_{n\leqslant x}\left(g(n)-\frac{1}{x}\sum_{m\leqslant x}g(m)\right)^2=O(xy),$$

so that from (14) and (17), the number of $n\leqslant x$ for which

(18) $\left|g(n)-\left(y-\frac{2y\log\log y}{\ldots}+\frac{c_5 y}{\ldots}\right)\right| \ldots \frac{y(\log\log y)^3}{\ldots}$

Carmichael's [illegible]

$$Q_T = \prod x \quad \text{and} \quad T = \prod x^{v_G(D)}$$

for our choice of D as $\lceil \nu/\log^3 \nu \rceil!$. Now let $l := \lceil \log \nu \rceil$ and consider the sum in

[illegible]

Note that

$$\sum_{j=0}^{[2w]} \frac{w^j}{j!} > \frac{e^w}{2} \quad \text{for} \quad w \geq 1.$$

Thus the quantity in (24) is greater than

Observe that m is relatively small: $m < (x^{1/y^3})^{y^2} = x^{1/y}$. Then by Theorem 2.5′ of it follows from the proof of Lemma 1 (i.e., from the fundamental lemma of the sieve) that

Since $r \leqslant v = \|j\|$, we see from (23) that

$$T_E \leqslant \log\log y + O(1).$$

Since the primes $p_1, p_2, \ldots, p_{r-1}$ already chosen satisfy (D), we see from (23)

In fact, we have

$$N(k) > \exp\left[\exp\left[(c_2 - o(1))\log k/\log\log k\right]\right] \tag{28}$$

for infinitely many k. On the other hand.