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## Carmichael's lambda function

by

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**1. Introduction.** Let  $\lambda(n)$  be the universal exponent for the group of residues mod  $n$  that are coprime to  $n$ . A more explicit definition of  $\lambda$  is:

$$\begin{aligned}\lambda(p^e) &= \phi(p^e) = p^{e-1}(p-1) && \text{if } p \text{ is an odd prime,} \\ \lambda(2^e) &= \phi(2^e) && \text{if } e = 0, 1, \text{ or } 2, \\ \lambda(2^e) &= \frac{1}{2}\phi(2^e) && \text{if } e \geq 3\end{aligned}$$

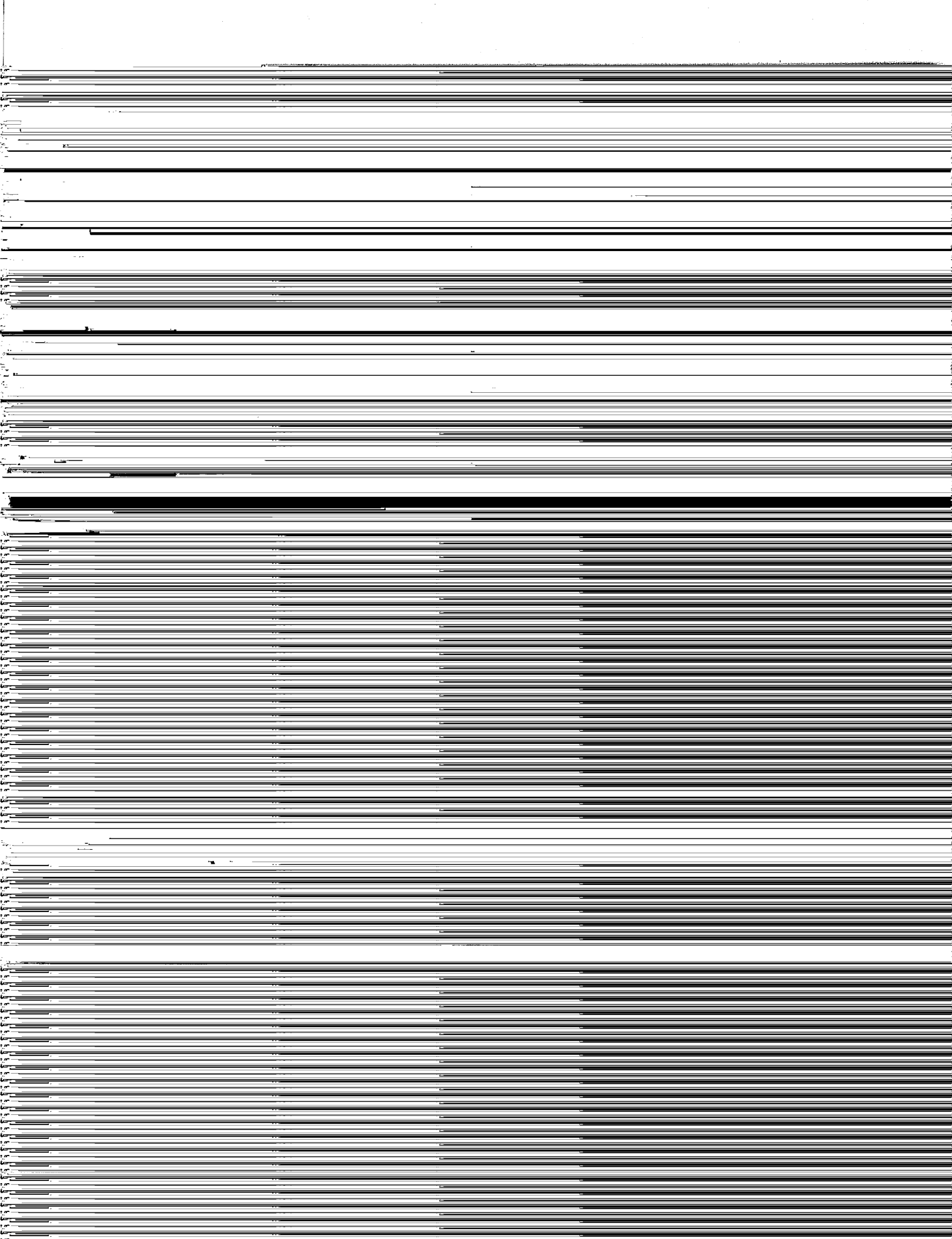
and finally,

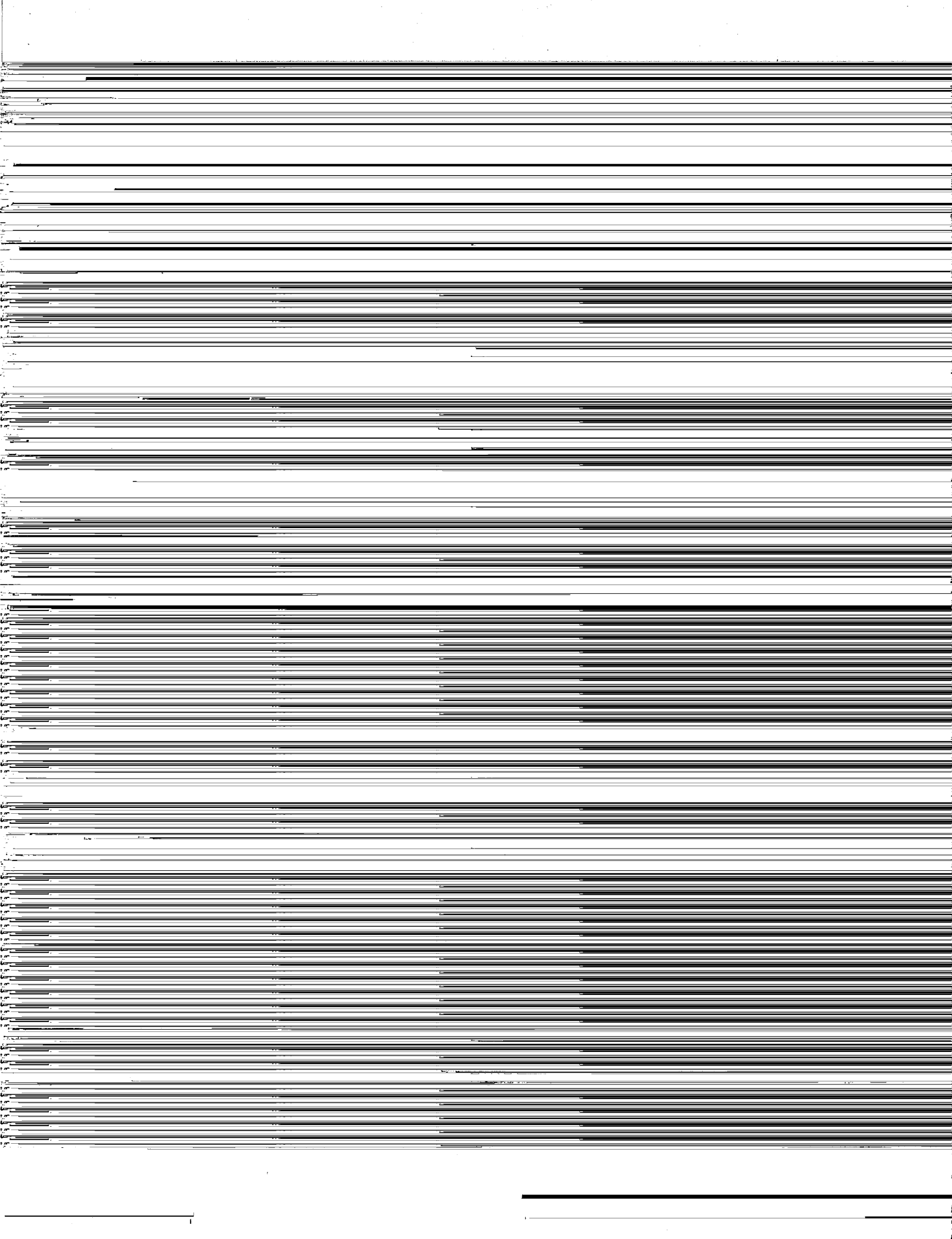


By taking a subsequence  $\langle n_i \rangle_i$ , we can obtain a sequence that is increasing and















From (15) we get

$$\sum_{n \leq x} \left( g(n) - \frac{1}{x} \sum_{m \leq x} g(m) \right)^2 = O(xy),$$

so that from (14) and (17), the number of  $n \leq x$  for which

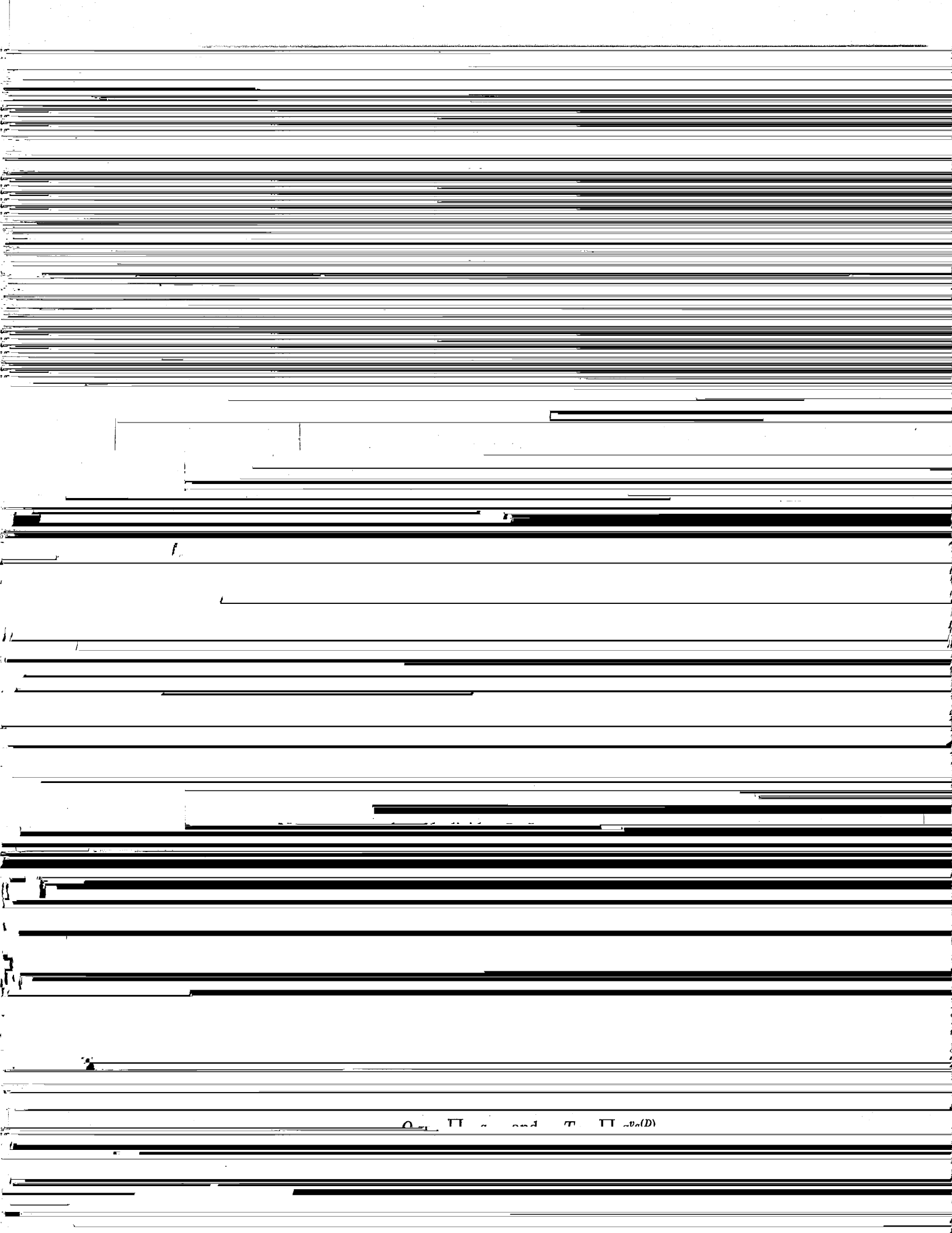
$$(18) \quad \left| a(n) - \left( \frac{2y \log \log y}{c_5 y} \right) \right| \geq y(\log \log y)^3$$

*Campylobacter* level 10.0



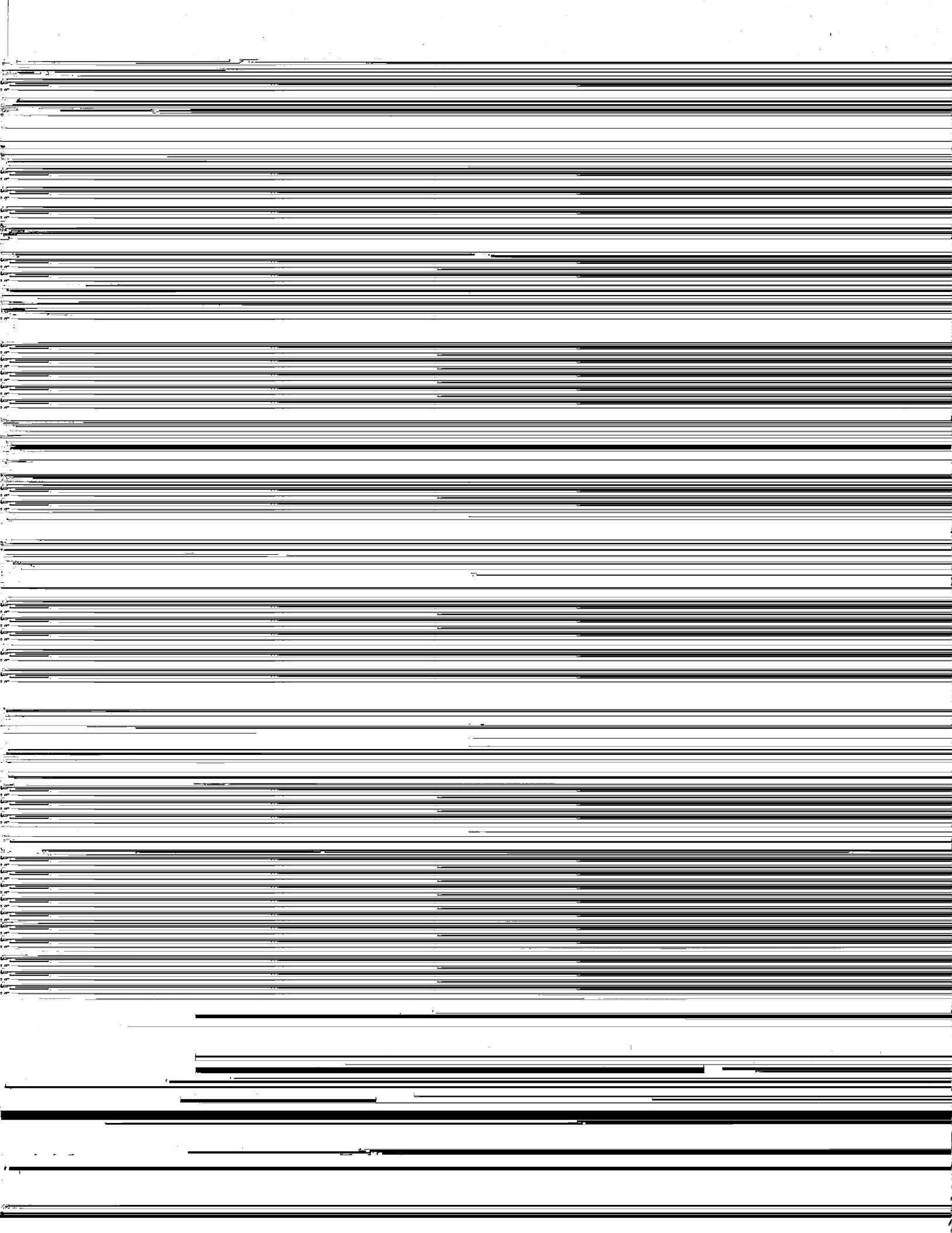






for our choice of  $D$  as  $\lceil v/\log^3 v \rceil$ . Now let  $l' = \lceil \log v \rceil$  and consider the sum in





Note that

$$\sum_{j=0}^{[2w]} \frac{w^j}{j!} > \frac{e^w}{2} \quad \text{for } w \geq 1.$$

Thus the quantity in (24) is greater than

Observe that  $m$  is relatively small:  $m < (x^{1/y^2})^{y^2} = x^{1/y}$ . Then by Theorem 2.5' of

it follows from the proof of Lemma 1 (i.e., from the fundamental lemma of the

sieve) that

Since  $r \leq v = \|j\|$ , we see from (23) that

$$T_E \leq \log \log y + O(1).$$

Since the primes  $p_1, p_2, \dots, p_{r-1}$  already chosen satisfy (D), we see from (23)

In fact, we have

$$(28) \quad N(k) > \exp[\exp[(c_2 - o(1)) \log k / \log \log k]]$$

for infinitely many  $k$ . On the other hand.

