

PRACTICAL SUGGESTIONS FOR MATHEMATICAL SPEAKING

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In the examples below, **red** is bad and **green** is good.

1. IMPORTANT THINGS

- (1) What is written on the board should be enough to stand alone as a coherent exposition.
- (2) Plan in advance what you will write on the board and what you will only say.
- (3) Write the little words like “so”, “**then**”, “**therefore**”, “**thus**”, “**because**”, “**on the other hand**” to make the logic clear.
- (4) When making a conditional statement, write “**If . . . , then . . .**” explicitly.
- (5) Always make the quantification (“for all”, “there exists”) clear. For example, instead of

An element a of a group G is in the center if and only if $ax = xa$.

write

An element a of a group G is in the center if and only if $ax = xa$ for all $x \in G$.

- (6) In a theorem statement, write “**Then**” to separate the hypotheses from the conclusions.
- (7) Distinguish statements from definitions. Here are three ways to do this on the board:
 - Write “**Def.**” when defining something.
 - Use $:=$ as needed. The statement “ $X = Y$ ” says that previously-defined objects X and Y are equal. In contrast, “**Let $X = Y$** ” and “ **$X := Y$** ” are two ways to express that X is being *defined* to equal Y .
 - Underline terms when they are being defined.

I sometimes use all three:

Def. The center Z of a group G is the set of elements of G that commute with all other elements of G :

$$Z := \{a \in G : ax = xa \text{ for all } x \in G\}.$$

- (8) Generally, the most important things are definitions (without that, you don’t what you are talking about), next are theorems and examples to illustrate them, and last are the proofs. So if you must skip something, skip the proofs (preferably the easy ones).
- (9) If you skip steps in a proof, tell the audience that you are doing this.
- (10) Write and say only true things! If you must say something that is not quite true (to simplify, to convey an idea without getting bogged down in details), then tell your audience that you are doing this.

Date: March 28, 2022.

The writing of this document was supported in part by National Science Foundation grants DMS-1601946 and DMS-2101040 and by Simons Foundation grants #402472 and #550033.

2. PREPARATION

- (11) If you will be speaking in an unfamiliar room, visit it beforehand so that you have a chance to compensate for any problems with it.
- (12) Find out long in advance what the size and blackboard configuration of the room is going to be, to help you decide whether to use blackboards, a video projector, or both. I generally prefer blackboards, because
 - In most setups, more material can be made visible at one time on blackboards than on a screen.
 - With prepared slides, there is a danger of going too quickly. For a 20-minute talk, I try to limit myself to about 10 slides.

But in certain situations (a huge lecture hall, a hotel conference room with only a tiny portable whiteboard, etc.), using a video projector might be the only workable option. Also, if you want to display a table or run a computer program in real time, then you might need a video projector for at least part of your talk.

3. STARTING AND ENDING

- (13) At the beginning, tell your audience what they are going to learn from your talk.
- (14) At the end, remind the audience what the main point was. What do you want the audience to take away from the talk?
- (15) Make it clear when your talk is over. One way to do this is to say
“Thank you for your attention.”

4. OTHER THINGS

- (16) If your normal speaking voice is soft, consciously speak more loudly than usual. Your voice needs to be clearly audible from the other end of the room.
- (17) If you appear interested in what you are talking about, the audience will be more interested too.
- (18) Some people speak without any notes in hand, but I think it is more important to have a talk go smoothly than it is to show off that I have it all in my head. I usually prefer to write out on paper in advance what I will write on the board; after doing so, however, I find that often I don’t need to consult the notes I am holding!
- (19) If presenting a paper, you probably will not have time to present everything in the paper. So think of your talk as an advertisement for the paper.
- (20) Sometimes working out a special case can be more illuminating than presenting a complete proof of the general case. In such a situation, present the special case!

5. IF USING A BLACKBOARD

- (21) Make sure that the chalkboard lights are on.
- (22) Check that there is enough chalk.
- (23) Erase the boards *completely*. (This applies also when you reuse a board during a talk. Small chalk marks left on the board can turn an f into an f' !)
- (24) Start writing in the upper left corner of each board.
- (25) If you write too small, the people in the back won’t be able to read it. If you write too large, you won’t be able to make much visible at a time.

- (26) If your handwriting is hard to read, try writing a little larger, or try slowing down a little to write your characters more carefully and completely.
- (27) Some rooms have stacks of vertically sliding blackboards. Often the back board in each stack cannot be moved, which means that after you write on it, you cannot use any other boards in the stack without covering it up. If you have such a stack, say with three boards, one strategy is
- start writing on the *middle* board, with the front board up;
 - next, raise the middle board and lower the front board and write on the front board;
 - finally, raise the front board and write on the back board.
- This way, each time you start a new board, the previous board is visible!
- (28) Finish filling up the left stack before moving to the right stack.

6. IF USING A VIDEO PROJECTOR

- (29) Prepare your slides using the $\text{\LaTeX}$ `beamer` package. The package is probably in your $\text{\LaTeX}$ distribution already; if not, download it from

<https://www.ctan.org/pkg/beamer>

The documentation is available here:

<http://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf>

For an example that you can use as a template, download the files

http://math.mit.edu/~poonen/slides/sample_beamer.tex

<http://math.mit.edu/~poonen/slides/increasing.png>

<http://math.mit.edu/~poonen/slides/zeta.png>

http://math.mit.edu/~poonen/slides/zeta_real.png

The main source file is the `.tex` file, and the `.png` files are image files that need to be in the same directory when you run $\text{\LaTeX}$ on the `.tex` file. The output should look like

http://math.mit.edu/~poonen/slides/sample_beamer.pdf

- (30) Because less material is visible on the screen at any given time than would be visible on multiple blackboards, remind the audience of definitions and your current goal as you go along.
- (31) Some people like to design a slide so that lines become visible incrementally, one at a time. But this aggravates the problem of not having enough material visible at a time. Personally I lean towards showing the whole slide at once, and I use incremental uncovering mostly when I want an element of surprise. Perhaps a compromise is to use the incremental style but have the upcoming text already half-visible in gray.
- (32) If you are stating theorems during your talk that are not yours, you should make an effort to give credit to the people who proved them and perhaps list the year too, by writing something like

`\begin{theorem}[Robinson 1949] \end{theorem}`

in your `.tex` file. Often in math talks speakers will not write their own name next to their theorems, however, and instead will write just the last initial:

`\begin{theorem}[P. 2019] \end{theorem}`

- (I guess this is some weird display of false modesty.)
- (33) A talk does not need to end with a bibliography slide full of references. Displaying the full bibliography of a paper is not very helpful for people listening to a talk because they won't be able to copy the information down anyway. On the other hand, if you want to recommend one to three references for further reading, maybe just author and title, then perhaps that might be useful to some people. I usually don't include such a list myself.
- (34) Some other options for how to end your slides:
- Display a slide summarizing the main takeaways from your talk.
 - Display a slide with further questions that have not been answered.
 - Design your last slide with information that will prompt audience questions and discussion, not just “Questions?” or “Thank you”.
- (35) During the talk, keep lights on if the slides can still be read that way, so that people don't get sleepy. If someone says “People will see the slides better if we turn the lights off”, then reply “It's extremely hard to see slides through closed eyelids” (suggestion of Patrick Winston).

7. NITPICKS

- (33) Do not use the symbol $\implies$ to mean “so” or “thus” or “then”. It is better to write the word, and to reserve $\implies$ for when you really want to talk about the logical implication (e.g., that X implies Y , without saying whether X and Y are true or not).
- (34) Consider underlining “Lemma”, “Prop.”, “Def.”, “Thm.”, “Cor.”, and the like to make these formal environments stand out.

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