

fa1

~~N469~~ → 1813
~~N645~~ 85
~~N808~~ 898

Neil Sloane, Esqr.
 Bell Laboratories
 Murray Hill, New Jersey 07974

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Dear Neil:

Lest I forget, here is an identification of sequence 645 (Lucas): Sequence 469 is the sequence for self-conjugate permutations (or the solutions of the telephone switching problem R1, 85 ff)

i.e. of

$$T_n = \sum_k \binom{n}{2k} a_k, \quad a_k = (2k-1)a_{k-1} = 1 \cdot 1 \cdot 3 \dots (2k-1)$$

$$= T_{n-1} + (n-1)T_{n-2}$$

N469
 (Leave fully)

If

$$T_n = (T+T)^n = \sum_k \binom{n}{k} T_k T_{n-k}$$

then

$$T_n = \sum_k \binom{n}{2k} a_k 2^{n-k} = 2(T_{n-1} + (n-1)T_{n-2})$$

Recurrence for N645

is sequence 645. Values are

n	0	1	2	3	4	5	6	7	8	9	10
T _n	1	2	6	20	76	312	1384	6512	32400	168992	921184

Incidentally $U_m = (T-T)^m = 0, m=2n+1$ $U_{2n} = 2^n a_n$

which is sequence 808, extended from n=5 to n=10.

It is clear that

$$T_n = C_n(2, 2, 0, \dots, 0) \quad - \text{cycle indicator}$$

In the telephone problem, the singlets are counted doubly (the local caller and the ^{outside} call he reaches) and so are the doublets. So T_n is the total number of people on the phone. But I still don't know what Lucas was counting.

Paul Stein was here last Friday and asks that you send him supplements (to the Handbook, of course!). Best as always and love to Ann.

John

N808

6
20
76
312
1384
6512
32400
168992
921184
5222208
30710464
186753920
1171979904
7573069568
50305536256
342949298688
2396286830080
17138748412928
125336396368896

End

N645

898

N808

? better descriptor ✓

$$(1.3.5 \dots (2n-1)) \times 2^n$$

$$\begin{aligned} 1.2 &= 2 & n=1 \\ (1.3) 4 &= 12 & n=2 \\ (1.3.5) 8 &= 120 \\ 105 \cdot 16 &= 1680 \end{aligned}$$

$$105 \cdot 16 = 1680 \quad \begin{array}{r} 3360 \\ 9 \\ \hline 30240 \\ 35 \end{array}$$

$$A=2 \quad B=1$$

$$F\phi R(N=2, N\leq 25; ++N)\{$$

$$B = \cancel{2 \cdot A} B + 2$$

$$A = \cancel{2 \cdot A \cdot B} 2 \cdot A \cdot B$$

$$A \}$$

run out, but looks good

$$[1.3.5 \dots (2N-1) \cdot 2^{**N} \cdot REF]$$

N645 - Extend

$$T_n = 2 \left(T_{n-1} + (n-1) T_{n-2} \right)$$

$J_C \quad J_B \quad J_A$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & \dots \\ 1 & 2 & 6 & 20 & 76 & \dots \end{array}$$

$$\text{Call } F93(1, JA)$$

$$\text{Call } F93(2, JB)$$

$$D\phi \mid N=1, \dots 30$$

$$\text{Call } M91(JA, N-1, JA)$$

$$\text{Call } A9(JA, JB, JA)$$

$$\text{Call } M91(JA, 2, J_C)$$

$$\text{Call } V9(JB, JA)$$

$$\text{Call } V9(J_C, JB)$$

$$\mid \text{Call } P9(J_C)$$