

being purchased

for

1870
1871
1629

July 23, 1968

Neil:

Enclosed: short table of Bessel polynomials
" " convolved Fibonacci nos.

For $p_n(x) = \sum_{k=0}^n \binom{n+k}{2k} x^k$ = assoc. Legendre polynomial
 $p_n(1) = f_{2n}$
 $p'_n(1) = \frac{1}{2} f_{2n-1}^{(2)}$ - convolved Fib no of order 2 - (*)

$$\pi_n(x) = \sum_{k=0}^n p_k(x) = \sum_{k=0}^n \binom{n+1+k}{2k+1} x^k$$

$$\pi_n(1) = f_{2n+1}$$

$$\pi'_n(1) = \frac{1}{2} [f_{2n}^{(2)} - f_{2n+1}^{(2)}] = f_{2n+1}^{(2)} - (n+2)f_{2n+1} = \sum_{k=0}^n p'_k(1)$$

$$= \pi'_{n+1}(1) - p'_n(1)$$

n	0	1	2	3	4	5	6	7	8	9	10	
$p'_n(1)$	0	1	5	19	65	210	654	1985	5911	17345	50305	1870
$\pi'_n(1)$	0	1	6	25	100	300	954	2929	8840	26185	76490	1871
$u'_n(1)$	0	0	1	2	5	10	20	38	71	130	235	1629

type 1 (*)

many
error

P.S.

For $u_n = \sum_{j=0}^{\lfloor n/2 \rfloor} \binom{n-j}{j} x^j$ = assoc Chebyshev polynomial

$$u_n(1) = f_n$$
 - Fibonacci $f_0 = f_1 = 1$

$$u'_n(1) = f_{n-2}^{(2)} = \sum_{j=0}^{\lfloor (n-2)/2 \rfloor} \binom{n-2-j}{j} = \sum_{j=0}^{\lfloor (n-1)/2 \rfloor} \binom{n-2-j}{j} = u_{n-2} - u'_{n-2}(1) \checkmark$$

VR

Bessel Polynomials

$$y_n(x) = \sum_{k=0}^n \binom{n+k}{2k} a_k x^k = (2n-1)x y_{n-1} + y_{n-2}$$

$$a_k = (2k)! / 2^k k! = (2k-1)!!$$

1515
1147
457
1879

1881
906
907

✓ 1515		y_{nk}											
y_{nk}	n/k	0	1	2	3	4	5	6	7	8	9	10	
1	0	1											1
2	1	1	1										2
7	2	1	3	3									3
87	3	1	6	15	15								4
266	4	1	10	45	105	105							5
2931	5	1	15	105	420	945	945						6
27007	6	1	21	210	1260	4725	10395	10395					7
353522	7	1	28	378	3150	17325	62370	135135	135135				8
5329837	8	1	36	630	6930	51975	270270	945945	2027025	2027025			9
90960751	9	1	45	990	13860	135135	945945	4729725	16216200	34459425	34459425		10
	10	1	55	1485	25740	315315	2837835	18918900	98891800	310134825	654729075	654729075	11
													12
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													20
													21
													22
													23
													24
													25
													26

Constant differences
in this direction

1881

1880

457

1879

1147

1515
1147
457
1879