Scan A2083
etc
&
A37254
Wynne & Narayana
Preprint
(it was published)

add to 6 sequences

TOURNAMENT CONFIGURATION, WEIGHTED VOTING, AND PARTITIONED CATALANS

by

B. E. Wynne and T. V. Narayana

1. Introduction Integer sequences can serve as powerful explicit descriptions of fundamental processes underlying organized activities. Seemingly unrelated decision processes may be perceived as mere variants of one another, once defined in terms of their essential

sequences. As an example, two diverse sporting and/or business activities will be shown to involve the same recursively describable integer sequence.

One of the processes is key to determining the champion in a match play tournament. The second process considered is that of decision-making by voting. Following a brief exposition of each

process and of the sequence they both utilize, some additional points of process similarity are noted as questions worthy of further investigation.

2. Match Play Tournaments As in [1], define T as the total number p is the number of players, if any, who may be accommodated in an incomplete "second flight." Letting k represent the number of rounds in such a tournament, we have; $\lceil (n-1)/2 \rceil \leq k \leq (n-1)$.

Capell and Narayana [1] showed that the number of different tournament match play arrangements possible for n entrants and a feasible number of match play rounds, k, is given by;

$$(1) \quad T_{(n,k)} = \sum_{i=\lceil (n-1)/2 \rceil}^{n-1} T_{(i,\, k-1)}$$

Though given in a slightly different form, their results of summing $T_{(n,k)}$ across the admissible range on k can be stated as:

$$(2) \quad T_n = \sum_{k=1}^{n-1} T_{(n,k)}$$ with slight notational change.

Capell and Narayana concluded by presenting the data given in Table 1. That data gives for n, the number of entrants, the number of tournaments or possible match-tree configurations, T_n, for all admissible k values.

- -

Insert Table 1. about here.

- -

The essence of that methodology was to recursively compute a vector element increment, I_m, as follows;

$$(3)\quad I_m = 2(I_{m-1}) - \text{mod}_2(m-1)\ (I_{[m/2]-1})$$

where, $m \geq 3$ and, $I_0 = 0$, $I_1 = I_2 = 1$.

Then, given the integer weighting elements, $W_{.j}$, of the weights vector, $[W]_{m-1}$, the elements of the vector $[W]_m$ are given by:

$$(3)\quad W_{m,j} = W_{m-1,j} + I_m$$

where $j = 1,\ldots,m-1$ and, $W_{m,m} = I_m$.

Finally, denoting the sum of the weights vector elements as S_m, the data of Table 2 is generated for illustration. Comparison [illegible]

n m

closes the same sequence of numbers. That sequence is #297, p. 53 of [4]. This equivalence, for $m = (n-1)$, is perhaps more readily [illegible]

[illegible] in (3).

Letting, $I_0 = 0,\ I_1 = I_2 = 1$

then $S_n = nI_n + (n-1)\,I_{n-1} + \ldots + 1$

from which $S_n - S_{n-1} = nI_n$.

Also, $W_{n,n-k} = I_n + I_{n-1} + \ldots + I_{n-k}$ for $k = 1, \ldots, n-1$

so that $\sum_{x=1}^{y} W_{n,x} < \sum_{x=n-y}^{n} W_{n,x}$ for $y=1, \ldots, \frac{n-1}{2}$ and, $n>2$

thus satisfying the three required conditions:

[redacted]

larger weights' sum taken from the same weights vector

(b) no subset of $[W]_n$ Equals any other non-intersecting subset

(c) S_n is minimal.

- -

Insert Table 2. about here.

- -

[redacted]

either, (1) assuming that each voter has a unique initial

opinion and that the content of the final resolution adopted is not identical with any initial individual opinion, then there are actually m+1 "entrants" in the voting process.

or, (2) assuming that the tournament champion is analogous

to the voting decision, then the n-1 non-champion tournament entrants are in one-to-one correspondence with the members of the voting body.

Based upon either interpretation of the conceptual shift discussed above, both processes can be thought of as being described by a "planar planted tree structure" which is trivalent, or limited to

of such trees (a single root, n tips, and k branching levels, where k ranges from [(n-1)/2] to n-1). One interpretation [4, p. 19] of demonstrates that the I sequence given here is a count of the [redacted]

provided that polygonal rotations (or planar tree reflections, whether full or partial) are disallowed.

However, a curious point of difference exists between the partitioning used (1) on trees counted as a subset of Euler's triangularization, and (2) those counted as match play tournament configura- [redacted] k branching levels for a [redacted]

given value of n tips as one class. In contrast, the Euler subset method counts trees in a manner ambiguous as to k, but correct overall for an n forest. This ambiguity occurs if one attempts to designate which of the n+1 endpoints serves as the root. In general, one triangu- [redacted] larized polygon can represent several different branching level values.

A caution is in order, having suggested that this Catalan subaggregation of coalitions. In some instances a board decision is formed "rapidly", just as a tournament with a minimum of byes may be said to progress rapidly. In other instances some individuals may fail to form or join any coalition until late in the board decision process. This is analogous to a sports tournament with byes approaching the maximum. Preserving prior terminology, it may be said that the kth coalition in effect determines the board's decision.

Two conjectures remain to be made in the context of the Board of Directors problem. First, the S_n sequence of Table 1 does not appear in Sloane's collection. However, Riordan [6, p. 215] discusses what he terms reciprocal central factorial numbers as being

TABLE 1.

Match Tournament Entrants and Configurations

Entrants, n	2	3	4	5	6	7	8	9	10	11
Configurations, T	1	1	2	3	6	11	22	42	84	165

A2083

TABLE 2.

Minimal-sum, Non-distorting, Tie-avoiding Integer Vote Weights

members, m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
totals, S_m	1	3	9	21	51	117	271	607	1363	3013	6643	14491	31495	67965	146115
column vector of vote weights, W_m	1	2	4	7	13	24	46	88	172	337	667	1321	2629	5234	10444
		1	3	6	12	23	45	87	171	336	666	1320	2628	5233	10443
			2	5	11	22	44	86	170	335	665	1319	2627	5232	10442
				3	9	20	42	84	168	333	663	1317	2625	5230	10440
					6	17	39	81	165	330	660	1314	2622	5227	10437

Δ = A37354

References

[1] P. Capell and T.V. Narayana, "On Knock-out Tournaments", Canadian Mathematical Bulletin 13 (1), 1970, pp. 105-9.

[2] D.L. Silverman, "The Board of Directors' Problem", Journal of Recreational Mathematics 8 (3), 1975-76, p. 234.

[3] B.E. Wynne, "INTWAT, STREAK and Weights Vectors", submitted May 15, 1976 to Journal of Recreational Mathematics.

[4] N.J.A. Sloane, A Handbook of Integer Sequences, Academic