

FACULTY OF ARTS AND SCIENCE / DEPARTMENT OF MATHEMATICS

June 24, 1968

A 2620 A 7590 A 11971 A 40027 A 38561

111 A

A 1040-A1110 A1350

A-1358

Dear Neil,

Professor Neil J.A. Sloane,

Cornell University, ITHACA, N.Y. 14850.

School of Electrical Engineering,

I was glad to meet you at Wisconsin, and I value your enthusiastic list of sequences.

I have jotted down a few items which you might like to consider for inclusion in a later draft (or which may already be there: I am not yet very skilled at using your list). I am sorry I haven't been able to devote more time to this as I think you have made a very substantial start to a worth-while idea, which others have thought of, but, as far as I know, not had the strength of purpose to put into action. Other



sequences S.T and t, might be of interest, but although S and T are infinite, we conjecture that they contain only a finite number of non-zero terms! Topological properties of graphs. On checking the records, I find I haven't sent you research papers #44 or 50, either, and this reminded me of the heading. I see you have the thickness of the complete graph. The genus is $n = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22$ $\gamma(K_{\alpha}) = 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 8 \ 10 \ 11 \ 13 \ 16 \ 18 \ 20 \ 23 \ 26 \ 29$ This is the Heawood conjecture recently completely established by Ringel and Youngs. The general formula is the least integer not less than (n-3)(n-4)/12. There is also the converse formula of Heawood for $\chi(p)$, the integer part of $\frac{1}{2}(7 + \sqrt{1 + 48p})$, which starts (for $p = 0,1, \ldots$) 4?,7,8,9,10,11,12,12,13,13,14,15,15,16,16,16, 17,17,18,18,19, ... except that the first term may be 5 if the 4color conjecture is false! Perhaps you should include two sequences The crossing number of the complete graph I notice you have (probably further than it is known--I am unable to pin down the

Berlekamp's numbers. Mentioned in his talk at Madison on Block coding for the symmetric channel with noiseless feedback, I just quote the table from my notes:

 1	1	1	1	1	1	1	2	4
 0	0	0	0	0	1	4	6	8
 0	0	0	1	5	10	14	22	36
 0	1	6	15	24	36	58	94	1.52
 7	21	39	60	94	152	246	398	644
 60	99	154	246	398	644	1042	1686	2728
 253	400	644	1042	1686	2728	4414	7142	11556

The numbers in the third column refer to $0.3,6,9,12,15,18,\ldots$ questions (and I think the first two columns to numbers of questions which are congruent to 1 or 2, modulo 3). Most of the entries are the sum of those a bishop's and a knight's move away to the left and up. I don't think he has a general formula. Write to him about them.

Bell numbers. You have these, but Leo Moser showed me a generalization at Los Angeles last March. "Aitken's array" (he called it) is

Start with 1.1 and add placing 2 below first 1 and to right of

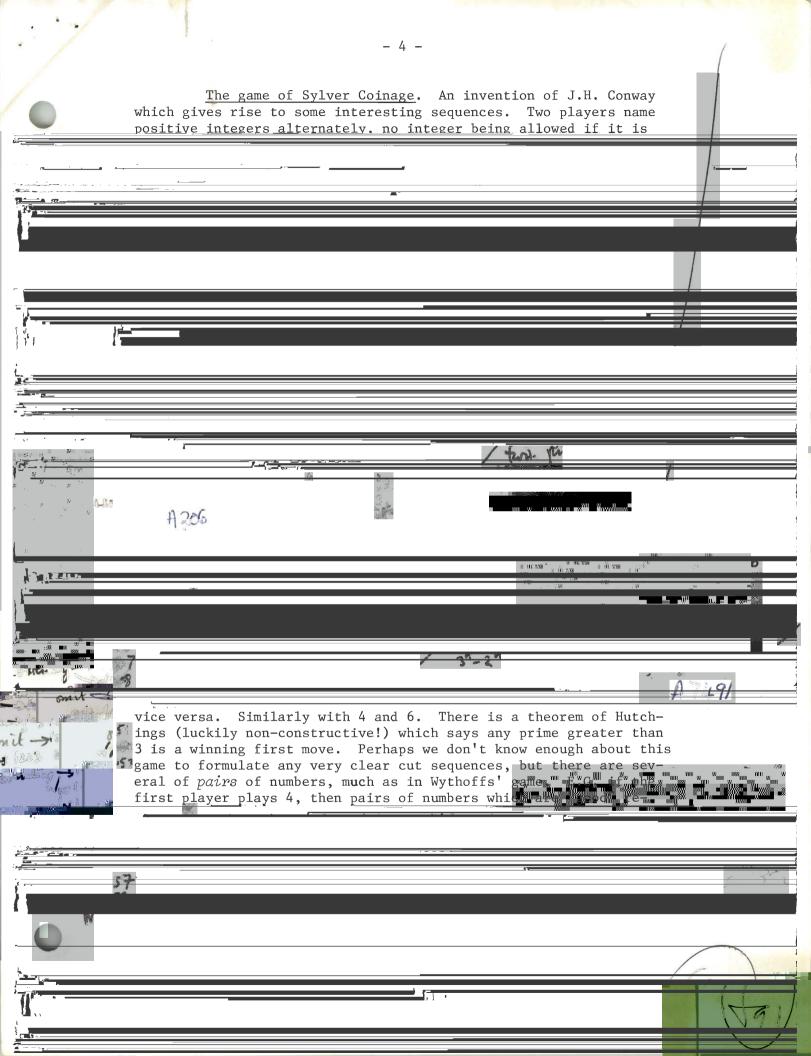
Now keep adding consecutive pairs in any row, placing

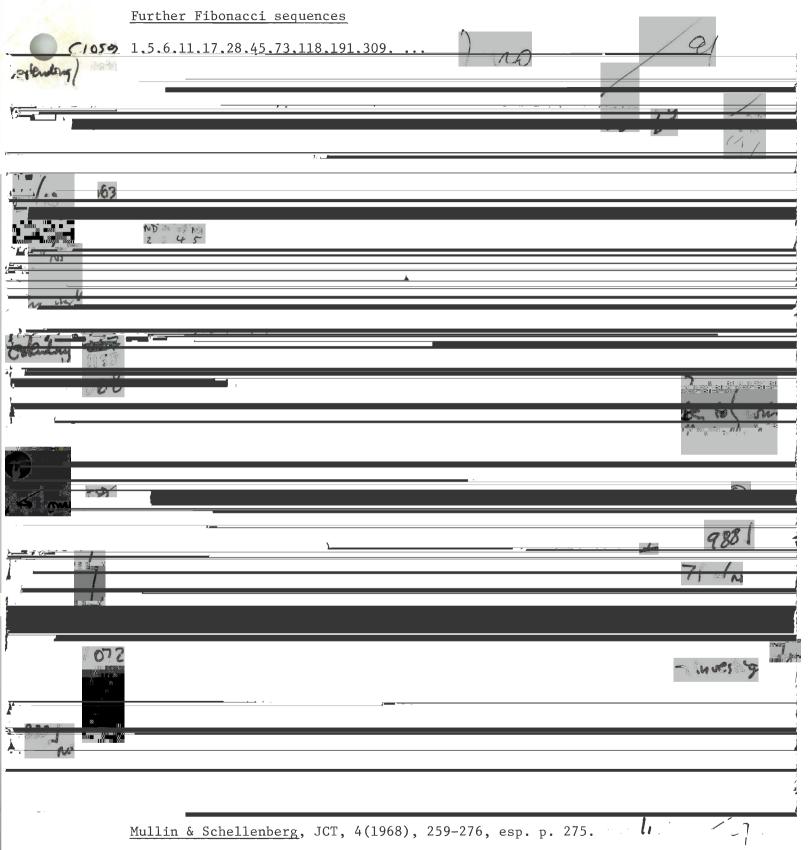


line when you run our of pairs to add. If you start with 0,1 in/ place of 1,1; Leo called them Bell numbers also, I believe, 0,1/1, $3,9,31,121,523, \ldots$ and gave the generating function

$$e^{e^x} \int_{0}^{x} e^{e^{t}} dt = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!}.$$







Miscellaneous



All less proper

Grey - Jh

191



FACULTY OF ARTS AND SCIENCE / DEPARTMENT OF MATHEMATICS



August 6, 1968

(to be mailed aft



Dr. N.J.A. Sloane, School of Electrical Engineering, Phillips Hall, Cornell University, ITHACA, N.Y. 14850.

Dear Neil,

