THE UNIVERSITY OF CALGARY

FACULTY OF ARTS AND SCIENCE / DEPARTMENT OF MATHEMATICS

June 24, 1968

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A1350
A1358

Professor Neil J.A. Sloane,
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ITHACA, N.Y. 14850.

Dear Neil,

I was glad to meet you at Wisconsin, and I value your enthusiastic list of sequences.

I have jotted down a few items which you might like to consider for inclusion in a later draft (or which may already be there: I am not yet very skilled at using your list). I am sorry I haven't been able to devote more time to this as I think you have made a very substantial start to a worth-while idea, which others have thought of, but, as far as I know, not had the strength of purpose to put into action. Other sequences S,T and t_n might be of interest, but although S and T are

infinite, we conjecture that they contain only a finite number of non-zero terms!

<u>Topological properties of graphs</u>. On checking the records, I find I haven't sent you research papers #44 or 50, either, and this reminded me of the heading. I see you have the thickness of the complete graph. The *genus* is

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
$\gamma(K_n)$	0	0	0	0	1	1	1	2	3	4	5	6	8	10	11	13	16	18	20	23	26	29

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This is the Heawood conjecture recently completely established by Ringel and Youngs. The general formula is the least integer not

less than $(n-3)(n-4)/12$. There is also the converse formula of Heawood for $\chi(p)$, the integer part of $\frac{1}{2}(7 + \sqrt{1+48p})$, which starts (for p = 0,1, ...) 4?,7,8,9,10,11,12,12,13,13,14,15,15,16,16,16, 17,17,18,18,19, ... except that the first term may be 5 if the 4-color conjecture is false! Perhaps you should include *two* sequences

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The crossing number of the complete graph I notice you have (probably further than it is known--I am unable to pin down the

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Berlekamp's numbers. Mentioned in his talk at Madison on Block coding for the symmetric channel with noiseless feedback, I just quote the table from my notes:

4	2	1	1	1	1	1	1	1	...
8	6	4	1	0	0	0	0	0	...
36	22	14	10	5	1	0	0	0	...
152	94	58	36	24	15	6	1	0	...
644	398	246	152	94	60	39	21	7	...
2728	1686	1042	644	398	246	154	99	60	...
11556	7142	4414	2728	1686	1042	644	400	253	...
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	...
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	...

The numbers in the third column refer to 0,3,6,9,12,15,18, ... questions (and I think the first two columns to numbers of questions which are congruent to 1 or 2, modulo 3). Most of the entries are the sum of those a bishop's and a knight's move away to the left and up. I don't think he has a general formula. Write to him about them.

Bell numbers. You have these, but Leo Moser showed me a generalization at Los Angeles last March. "Aitken's array" (he called it) is

1	1	2	5	15	52	203
2	3	7				
5	10					
15						

δ = A1971

Start with 1,1 and add placing 2 below first 1 and to right of second. Now keep adding consecutive pairs in any row, placing [illegible]

line when you run our of pairs to add. If you start with 0,1 in place of 1,1; Leo called them Bell numbers also, I believe, 0,1,1, 3,9,31,121,523, ... and gave the generating function

$$e^{e^{x}} \int_{0}^{x} e^{e^{t}-1}\, dt = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!} .$$

The game of Sylver Coinage. An invention of J.H. Conway which gives rise to some interesting sequences. Two players name positive integers alternately, no integer being allowed if it is

…

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vice versa. Similarly with 4 and 6. There is a theorem of Hutchings (luckily non-constructive!) which says any prime greater than 3 is a winning first move. Perhaps we don't know enough about this game to formulate any very clear cut sequences, but there are several of *pairs* of numbers, much as in Wythoffs' game. E.G. if the first player plays 4, then pairs of numbers which are good re-

Further Fibonacci sequences

(1050 1,5,6,11,17,28,45,73,118,191,309, ...

extending)

[redacted]

Mullin & Schellenberg, JCT, 4(1968), 259-276, esp. p. 275.

Miscellaneous

M1

Other pairs of Lucas-Lehmer-Pell-Fibonacci sequences, e.g. M2

(a) 0,1,4,15,56,209,780,2911,10864,40545,151316, ... and

$A_{n+1} = 4A_n - A_{n-1}$

1,2,7,26,97,362,1351,5042,18817,70226,262087, ...

$A_{n+1} = 4A_n + A_{n-1}$

(b) 0,1,4,17,72,305,1292,5437,23184,98209, ... and
1,2,9,38,161,682, ...

$A_{n+1} = 10A_n - A_{n-1}$

(c) 0,2,20,198,1960,19402,192060,1901198,18819920, ... and
1,5,49,485,4801,47525,470449,4656965,46099201, ...

$A_{n+1} = 16A_n - A_{n-1}$

(d) 0,3,48,765,12192,194307,3096720,49353213, ... and A5068
1,8,127,2024,32257,514088,8193151,130576328, ...

6+

(e) 0,1,6,37,228,1405,8658,53353, 328776,2026009, ... and

20−

1,3,19,117,721,4443,27379,168717,1039681,6406803, ...

(f) 0,3,60,1197,23880,476403,9504180,189607197, ... and 36+

30−

1,10,199,3970,79201,1580050,31521799, ...

(g) 0,5,180,6485,233640,8417525,303264540, ... and 8−

1,18,649,23382,842401,30349818,1093435849, ... 8+

(h) 0,4,120,3596,107760,3229204,96768360, ... and
1,15,449,13455,403201,12082575,362070409, ... 10+

(i) 0,1,8,63,496,3905,30744, ... and 140+
1,4,31,244,1921,15124,119071, ...

12+

(j) 0,1,8,65,528,4289,34840,283009,2298912, ... and
1,4,33,268,2177,17684,143649,1166876,9478657, ... 64+

(k) 0,1,10,101,1020,10301,104030,1050601,10610040, ... and
1,5,51,515,5201,52525,530451, ...

364+

(l) 0,13,1820,254813,35675640,4994844413, ... and

16+

All been entered

Later w Guy - why?

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August 6, 1968

(to be mailed after strike)

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Dr. N.J.A. Sloane,
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Dear Neil,

Thank you for your letter of 9:7:68, which I received after getting back from a second visit to the [redacted]

Research Papers 11 and 12. The first contains a sequence [redacted]