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PROBLEMS AND SOLUTIONS

EDITED BY MURRAY S. KLAMKIN

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All problems and solutions should be sent, typewritten in duplicate, to Murray S. Klamkin, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1, and should be submitted in accordance with the instructions given on the inside front cover. An asterisk placed beside a problem number indicates that the problem was submitted without solution. Proposers and solvers whose solutions are published will receive 10 reprints of the corresponding problem section. Other solvers will

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$$H(2N, T) = \binom{\ }{V}\Big/\binom{\ }{U},$$

$$H(2N + 1, T) = \binom{2N}{2V}\Big/\binom{2N}{T - 1}$$

and

$$U = \left[\frac{T - 1}{2}\right], \qquad V = \left[\frac{T}{2}\right]$$

and $[x]$ denotes the greatest integer $\leqq x$.

Problem 75-2, *The Regiment Problem Revisited*, by G. J. SIMMONS (Sandia Laboratories).

Is it possible to form a marching column of two's with $n - 1$ members from each of n regiments in such a way that every regiment is paired with every other regiment and no two members of the same regiment have fewer than the obvious maximum-minimum of $[(n - 3)/2]$ ranks separating them?

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Problem 75-3, *A Power Series Expansion*, by U. G. HAUSSMANN (University of British Columbia).

Last year, a former engineering student of ours wrote to the mathematics department concerning a problem encountered in the electrical design for the appurtenant structures of the Mica Dam on the Columbia River in British Columbia. These structures include a spillway, low level outlets, intermediate level outlets, auxiliary service buildings and a power intake structure.

The engineers obtained a function

$$f(u) = [\exp(u + nu) + \exp(-nu)]/[1 + \exp u], \tag{1}$$

where

$$\cosh u = 1 + x/2 \tag{2}$$

and where x is a ratio of resistances. Moreover, they suspected that if $y = f[u(x)]$, then

$$y(x) = \sum_{k=0}^{n} \binom{n+k}{2k} x^k. \tag{3}$$

Show that (3) is valid.

Problem 75-4*, *A Combinatorial Identity*, by P. BARRUCAND (Université Paris VI, France).

Let

$$A(n) = \sum_{i+j+k=n} \frac{n!^2}{i!^2 j!^2 k!^2},$$

where i, j, k are integers $\geqq 0$, and let

$$B(n) = \sum_{m=0}^{n} \binom{n}{m}^3,$$

so that $A(n)$ is the sum of the squares of the trinomial coefficients of rank n and $B(n)$ is the sum of the cubes of binomial coefficients of rank n ($A(n) = 1, 3, 15, 93, 639, \cdots$, $B(n) = 1, 2, 10, 56, 346, \cdots$).

Prove that

$$A(n) = \sum_{m=0}^{n} \binom{n}{m} B(m).$$

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Editorial note. Equivalently, one has to prove that

$$\sum_{m=0}^{n} \sum_{r=0}^{m} \binom{n}{m}\binom{m}{r}^3 = \sum_{m=0}^{n} \binom{2m}{m}\binom{n}{m}^2.$$

For other properties, e.g., recurrences, integral representations, etc., the proposer refers to his papers in Comptes Rendus Acad. Sci. Paris, 258 (1964), pp. 5318–5320 and 260 (1965), pp. 5439–5541. He also notes that his solution is a tedious indirect one. [M.S.K.]