A3099

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H W Gould

letter

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7 pages

A(n+1) New seq

A589 5
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WEST VIRGINIA UNIVERSITY • MORGANTOWN, WEST VIRGINIA
COLLEGE OF ARTS AND SCIENCES
26506

DEPARTMENT OF MATHEMATICS

A3099

A[illegible]

A21

16 Nov. 1973

Dr. Neil J. A. Sloane,
Bell Laboratories,
600 Mountain Avenue,
Murray Hill, N.J. 07974

Dear Dr. Sloane,

I appreciated your letter of 7 November very much. I don't know whether at most 5 elementary transformations will do it, but a finite number will surely suffice! As for handshakes, I don't have the faintest idea. But I do maintain a voluminous correspondence which may help explain why I am only now finally getting around to sending you the table of Fibonomial Catalan numbers. I calculated these by hand (two ways, recursively and every tenth one being checked by direct calculation) while my wife and I watched science fiction (and Dracula, Frankenstein, etc.) Saturday nights [redacted] here from a Pittsburgh TV channel (sitting up till 3 or 4 [redacted] Makes the TV interesting when the plots are dull. The result is a ten-page table of the first 50 Fibonomial [redacted] Catalan numbers in factored form. The fiftieth one has 11 [redacted] distinct prime factors. [illegible]

Anyway, enclosed is a Xerox copy of the first two [illegible] I had no precise [illegible] for the actual digits in those beyond K(10) so that was the last one I gave that way. You can easily run off the values of others from the factors I give. These first two pages will give you K(n) for n=1(1)22. Be careful to note that where n is [illegible] given in the left hand col[illegible] then K(n+1) is the value given at [illegible] the right. I did this to agree with the usual [illegible] notation I use for ordinary Catalan numbers, $C(n) = \binom{2n}{n}/(n+1)$ [illegible]

A108

I [illegible]

[illegible] 109601. [illegible]

A522

[illegible]

A589 [illegible] = [illegible] = 2, [illegible]

A3149

and

New

$$S'_n = \sum_{k=0}^{n} k!(n-k)! - 1 = 2, 5, 16, 64, 312, 1812, 12288, 95616,$$

840960, 82 4080, 89 1280, 106 3 9 0, 13 4 6 2 0

You will notice that the two start alike, but begin to differ by 1 unit with n = 4, and then the difference grows, but not too remarkably. In fact it is easy to show that

[redacted]

Recurrences:

These two sequences have recently been confused with one another and I am writing a review for the German Zentralblatt für Mathematik (copy enclosed in fact) that shows how this has happened. I think a table of sequences such as yours would be of great help in just such situations.

Of course, the two sequences above are familiar in other notation ...

I came across S'_n one time years ago in calculating resistance of an array of resistors in electrical engineering work.

I would very much like to see your book. How nearly ready is it now? I saw a mention of a prepublication version in a recent paper

Best regards

Henry W. Gould

P. S.: I collect peculiar sequences, and might be able to suggest some others you do not have.

P. P. S.: I should remark that you be sure to know that I use F with this definition: $F_0 = 0$, $F_1 = 1$, then $F_{n+1} = F_n + F_{n-1}$

N 1248.5 = 3150

FIBONOMIAL CATALAN NUMBERS

[redacted] $1 \left(2k \right) = F_{2k}F_{2k-1} \cdots F_{k+2}$ [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

k	n	F_n	$K(k+1)$
	26	121393	
13	27	196418	$2\cdot3\cdot5\cdot7\cdot17\cdot19\cdot23\cdot37\cdot41\cdot47\cdot53\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot$ $\cdot1597\cdot3001\cdot28657$
	28	317811	
14	29	514229	$2^3\cdot5\cdot11\cdot17\cdot19\cdot23\cdot31\cdot37\cdot41\cdot53\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot$ $\cdot1597\cdot3001\cdot28657\cdot514229$
	30	832040	
15	31	1346269	$2^3\cdot5\cdot11\cdot17\cdot19\cdot23\cdot31\cdot37\cdot41\cdot53\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot$ $\cdot557\cdot2207\cdot2417\cdot3001\cdot28657\cdot514229$
	32	2178309	
16	33	3524578	$2\cdot5\cdot11\cdot23\cdot31\cdot37\cdot41\cdot53\cdot89\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot557\cdot$ $\cdot2207\cdot2417\cdot3001\cdot3571\cdot19801\cdot28657\cdot514229$
	34	5702887	
17	35	9227465	$2^2\cdot3^3\cdot5^2\cdot11\cdot13\cdot23\cdot31\cdot41\cdot53\cdot89\cdot107\cdot109\cdot199\cdot281\cdot421\cdot521\cdot$ $\cdot557\cdot2207\cdot2417\cdot3001\cdot3571\cdot19801\cdot28657\cdot141961\cdot514229$
	36	14930352	
18	37	24157817	$2^2\cdot3^2\cdot5\cdot13\cdot23\cdot31\cdot53\cdot73\cdot89\cdot107\cdot109\cdot149\cdot199\cdot281\cdot421\cdot521\cdot$ $557\cdot2207\cdot3221\cdot2417\cdot3001\cdot3571\cdot9349\cdot19801\cdot28657\cdot141961\cdot$

PRIME FACTORS OF CATALAN NUMBERS

$$C(k) = \frac{1}{k+1}\binom{2k}{k}$$

k	n	$\binom{n}{k} - \binom{n}{k-1}$	Prime factors
0	1	1	
1	3	2	2
2	5	5	5
3	7	14	$2\cdot 7$
4	9	42	$2\cdot 3\cdot 7$
5	11	132	$2^2\cdot 3\cdot 11$
6	13	429	$3\cdot 11\cdot 13$
7	15	1430	$2\cdot 5\cdot 11\cdot 13$
8	17	4862	$2\cdot 11\cdot 13\cdot 17$
9	19	16796	$2^2\cdot 13\cdot 17\cdot 19$
10	21	58786	$2\cdot 7\cdot 13\cdot 17\cdot 19$
11	23	208012	$2^2\cdot 7\cdot 17\cdot 19\cdot 23$
12	25	742900	$2^2\cdot 5^2\cdot 17\cdot 19\cdot 23$
13	27	2674440	$2^3\cdot 3^2\cdot 5\cdot 17\cdot 19\cdot 23$
14	29	9694845	$3^2\cdot 5\cdot 17\cdot 19\cdot 23\cdot 29$
15	31	35357670	$2\cdot 3^2\cdot 5\cdot 19\cdot 23\cdot 29\cdot 31$
16	33	129644790	$2\cdot 3\cdot 5\cdot 11\cdot 19\cdot 23\cdot 29\cdot 31$
17	35	477638700	$2^2\cdot 3\cdot 5^2\cdot 11\cdot 23\cdot 29\cdot 31$
18	37	1767263190	$2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 23\cdot 29\cdot 31\cdot 37$
19	39	6564120420	$2^2\cdot 3\cdot 5\cdot 11\cdot 13\cdot 23\cdot 29\cdot 31\cdot 37$
20	41	24466267020	$2^2\cdot 3\cdot 5\cdot 13\cdot 23\cdot 29\cdot 31\cdot 37\cdot 41$
21	43	91482563640	$2^3\cdot 3\cdot 5\cdot 13\cdot 29\cdot 31\cdot 37\cdot 41\cdot 43$
22	45	343059613650	$2\cdot 3\cdot 5^2\cdot 13\cdot 29\cdot 31\cdot 37\cdot 41\cdot 43$
23	47	1289904147324	$2\cdot 3^2\cdot 13\cdot 29\cdot 31\cdot 37\cdot 41\cdot 43\cdot 47$

$n = 2k + 1$, whence $\binom{n}{k} - \binom{n}{k-1} = \binom{2k+1}{k} - \binom{2k+1}{k-1} = \dfrac{(2k+2)!}{(k+1)!(k+2)!} = C(k+1)$.

Theorem: $k \geqslant 2$ and $2k+1 =$ prime $\Longrightarrow 2k+1 \mid C(k+1)$.

Calculated by H. W. Gould

REVIEW TO APPEAR IN ZENTRALBLATT FÜR MATHEMATIK:

226218

1972 Wb

ZENTRALBLATT FÜR MATHEMATIK

Date:

Kaucký, Josef:

In the first place, the (correct) value given in the published solution is

$$(1) \qquad D_n = \sum_{j=0}^{n-1} S_j \,, \quad \text{where} \quad S_j = \sum_{i=0}^{j} \frac{j!}{(j-i)!} \,,$$

which is incorrectly given in the present paper with

$$(2) \qquad S_j = \sum_{i=0}^{j} i!(j-i)! \,.$$

This circumstance may be due to the fact that S is incorrectly

printed in one place in the problem solution as

$$S_j = \sum_{i=0}^{j} i!/(j-i)!$$

and possible confusion of $i!/(j-i)!$ with $i!(j-i)!$.

The first few values (for $j = 0,1,2,\ldots$) of S_j as given by (1) are 1, 2, 5, 16, 65, 326, ... whereas the values given by (2) (as noted by the author) are 1, 2, 5, 16, 64, 312, ... and the

correct values of D_n, correspondingly, are 1, 3, 8, 24, 89, 415,....

In the second place, the derivation of the value of D

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FATAL ERROR * MISSING ROUTINE    AB         ,AME        AT INSERTED AT REFERENCES
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*YRUN = (BCD) USER/RAR

 6
 16
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 182550368000
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 44527447040000
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 2361827297364885504000
 54153994430726360268800
 1294972786374034472960000
 32303355215793434132480000
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3149

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 50 DO 1 N=2,50
*YRUN = (BCD) USER/T@RAR
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#10

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LOGIN: NJS

DC

11 13 19 29 37 41 47 113 199**********p
700550979746059
233 421 1597***p
10974447816819957428273 9

[
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7 13 19 23 29 37 41 47 61 113 199*************p
225577415478230998
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43462364742441315642531 56182

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28657 1597*421*199*113*61*46#7*41*37*29*23*19*13*7*2*p
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