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More on Automorphic Numbers

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	To Find the Automorphic Disds (i. s	e., the units' digit	s) to any Base B
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	there is some positive integer	(m) for	which $nx = my + 1$. Then " nx "
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To Develop the Coefficient of "B" in an Automorphic Number (i.e. the second digit from the

right)

Square the automorphic digit. The right-hand digit will, of necessity, be the same

as the automorphic digit. Let the left-hand digit be s₁. Double the automorphic

()_

digit and let u be the units' digit (Modulus B) of this product. If a_1 is the second digit from the right of the automorphic number it is necessary and sufficient to

$$a_1 = (By - s_1) / (u - D) = 0$$

 a_1 and y being positive integers and $a_1 < B$. (No in If u = 0, then v = 0 and

stablis

 $a_1 = s_1$.

The second digit of the other automorphic nu ____ the pair a,' can be ob-

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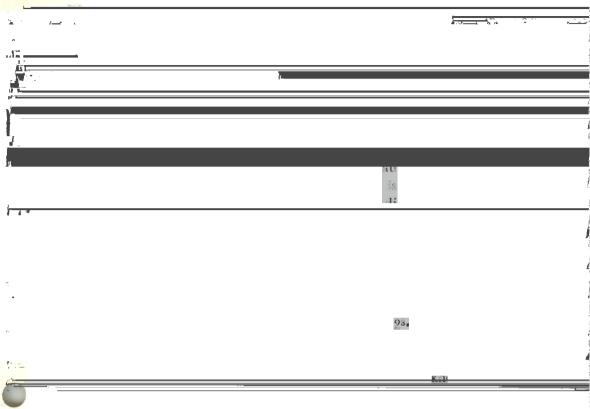
a' = B - 1 - a.

For example, if B=24, the automorphic digits (A and A') are "9" and "16". Consider "9" (in base 24):

$$9^2 = 3.09$$
: $s_1 = 3$; $u = 2.9 \pmod{24} = 18$; $a_1 = (24y - 3) / (18 - 1)$ $(a_1 < 24)$; $y = 15$; $a_1 = 21$; $a_1' = (24 - 1) - 21 = 2$

Therefore the same 12 in the same 1 0 10

For example: $(21-09)^2 = 00-21-09 \pmod{24^3}$, so $s_2 = 0$, $a_2 = 0$; $(2-16)^2 = 07-02-16 \pmod{24^3}$, so $s_2' = 7$, $a_0' = 23$. Therefore the three-digit automorphic



Note: Either one can be obtained from the other (0=24-1-23), but it is advisable to check the work as development progresses. Even with expansion factors, automorphic numbers can be developed only one digit at a time.

Some Observations

Automorphic digits must be greater than \sqrt{B} to eliminate trivial values. For every base B, s_1 increases as A increases. Also s_1 is always less than A.

When B is even but not divisible by A, there is one A itomorphic number whose

TABLE 1 (Continued)

<u>. </u>	Scale of	Automorphic.	Expansion		<u>v-</u>	l
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TABLE 2—Automorphic Numbers

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