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More on Automorphic Numbers

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Tables showing

[redacted]

non-trivial automorphic numbe

ber to give a general method of determi

(or automorphic digits) for any base

[redacted]

To Find the Automorphic Digits (i. e., the units' digits) to any Base B

[redacted]

umber of ways of reso

hich are prime to each oth

mber.)

ith $1 < m$ ing relatively

there is some positive integer $< m$) for which $nx = my + 1$. Then "nx"

automorphic digit and "$B +$ nx" is the other member of the pair. By

all relatively prime values of nd n, the total number of pairs of automo

anc

[redacted]

ations. In this case

[redacted]

digits.

one of the A" and th

[redacted]

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To Develop the Coefficient of "B" in an Automorphic Number (i.e. the second digit from the right)

Square the automorphic digit. The right-hand digit will, of necessity, be the same as the automorphic digit. Let the left-hand digit be s_1. Double the automorphic digit and let u be the units' digit (Modulus B) of this product. If a_1 is the second digit from the right of the automorphic number it is necessary and sufficient to

[redacted]

$$a_1 = (By - s_1) / (u - 1)$$

a_1 and y being positive integers and $a_1 < B$. (Note: If $u = 0$, then $y = 0$ and $a_1 = s_1$).

The second digit of the other automorphic number of the pair a_1' can be ob-

[redacted]

$$a' = B - 1 - a.$$

For example, if $B = 24$, the automorphic digits (A and A') are "9" and "16". Consider "9" (in base 24):

$9^2 = 3{\cdot}09$: $\quad s_1 = 3$; $\quad u = 2 \cdot 9 \pmod{24} = 18$;
$a_1 = (24y - 3) / (18 - 1)$ $\quad (a_1 < 24)$; $\quad y = 15$;
$a_1 = 21$; $\quad a_1' = (24 - 1) - 21 = 2$

[redacted]

For example: $(21\text{-}09)^2 = 00\text{-}21\text{-}09 \pmod{24^3}$, so $s_2 = 0$, $a_2 = 0$; $(2\text{-}16)^2 = 07\text{-}02\text{-}16 \pmod{24^3}$, so $s_2' = 7$, $a_2' = 23$. Therefore the three-digit automorphic

[redacted]

Note: Either one can be obtained from the other $(0 = 24 - 1 - 23)$, but it is advisable to check the work as development progresses. Even with expansion factors, automorphic numbers can be developed only one digit at a time.

Some Observations

Automorphic digits must be greater than $\sqrt{B}$ to eliminate trivial values. For

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every base B, s_1 increases as A increases. Also s_1 is always less than A.

When B is even but not divisible by 4, there is one automorphic number whose

[redacted]

TABLE 1 (Continued)

Scale of	Automorphic	Expansion

TABLE 2—Automorphic Numbers

Scale of	*Squares*	*Scale of*	*Squares*
[illegible]	[illegible]	[illegible]	[illegible]