Joe Roberts, Lure of the Integers. A7415

where the a, b, c, d are integers, $ad - bc \neq 0$, and $c \neq 0$. Given a value, say x_0, we can use T to define a sequence by putting $x_{n+1} = T(x_n)$. Two cases in which the x_i are all integers are afforded by:

1. $x_1 = 2$, $x_{n+1} = \dfrac{x_n}{x_n - 1}$, where the sequence is 2, 2, 2, ...

2. $x_1 = 4$, $x_{n+1} = \dfrac{178x_n - 1492}{7x_n + 2}$, where the sequence is 4, −26, 34, 19, 14, 10, 4, ...

Independently of the other parameters, if all the x_i are integers, then the sequence $\{x_i\}$ is periodic and always has period length one of 1, 2, 3, 4, or 6.

Neil, I thought that you did not allow short periodic seq.s!

REFERENCES

D. M. Adelman, Note on the arithmetic of bilinear transformations, *Proc. Amer. Math. Soc.* 1 (1950) 443–48.

I killed it

8 Polyhedra

In *A Survey of Geometry,* Howard Eves proves that every polyhedron may be dissected into a finite number of tetrahedra. In doing this the vertices of the tetrahedra need not all be vertices of the original polyhedron. On the other hand, if it is possible to so select the tetrahedra so that all of their vertices are vertices of the original polyhedron, then Eves calls the polyhedron a *Lennes polyhedron* .

In 1911 Lennes constructed a Lennes polyhedron with 7 vertices. In 1928 Schönhardt gave an example of such a polyhedron with 6 vertices and showed that none existed with fewer than 6 vertices. In 1948 Bagemihl showed Lennes polyhedra exist for any number of vertices greater than 6.

Thus, Lennes polyhedra of n vertices exist if and only if $n \geq 6$.

7416

fa4 ~~Seq~~, From Roberts p. 86

Let $A(h)$ = least pos. integer with exactly h divisors. From AS1 p. 840 :-

h	1	2	3	4	5	6	7	8
$A(h)$	1	2	4	6	16	12	64	24
9	36							
10	48							
11	102k							
12	60							
13	4096							
14	192							

(Seq. A 5179)

$\sigma_0(n)$ = # divisors of n

$A(h) = \min\{n : \sigma_0(n) = h\}$

Call n $\boxed{\text{minimal}}$ if

$$A(\sigma_0(n)) = n$$

Minimal numbers:

1, 2, 4, 6, ~~16~~ 12, 16, 24, 36, ...

it is Sequence A5179 sorted into increasing order, I think

594

A5179

→ A7416

```
20   m = 2  @ N = 0  @  S$ = "
30   N = N + 1
40   T = Nbr Div (n)
50   If  T = m  then  S$ = S$ & STR$ (N) & ','
                  @ S$ @ m = m + 1
60   Goto 30
```

2, 4, 6, 16, 18, 64, 66, 100, 112,

```
20   m = 2  @  S$ = "
25   N = 0
30   N = N + 1
40   T = Nbr Div (N)
50   If  T = m  then
                  @ S$ @ m = m + 1 @ Goto 25
60   Goto 30
```

2, 4, 6, 16, 12, 64, 24, 36, 48   = m 0940

A5179

A 74/6

10 ! M 0937

20 Option Base 1 @ DIM S[1000] N=1

30 N=N+1

40 T= NbrDiv(N)

50 If T>1000 then 30

~~60 if NOT S(T) then~~

~~70 Goto~~

60 if S(T) Then 30

70 Beep @ S(T)=N @ Disp '"'&STR$(T)&'"=' & STR$(N) @ Beep @ Goto 30

~~Assign #1 to~~ NN Data

Read #1; S()

ASC Sort #1

Assign #1 to "*"

A7416

n	d()
1	1
2	2
4	3
6	4
12	6
16	5
24	8
36	9
48	10
60	12
64	7
120	16

1, 2, 4, 6, 12, 16, 24, 36, 48, 60, 64, 120, 144, 180, 192,

240, 360, 576, 720, 840, 900, 960, 1024, 1260, 1296,

1680, 2520, 2880, 3072, 3600, 4096, 5040, 5184,

6300, 6480, 6720, 7560, 9216, 10080, 12288, 14400,

15120, 15360, 20160, 25200, 25920, 27720, 32400

36864,

39563,76805,149360,290896,567321,1107775,2165487,4237384,8299283
Related to population of numbers of form x^2+y^2. Ref MOC 18 84 64. [1,2; A0694, N0384]

M0937 1,2,4,6,12,16,24,36,48,60,64,120,144,180,192,240,360,576,720,840,
900,960,1024,1296,3072,4096,9216
Minimal numbers: n is smallest munber with this number of divisors. Cf. M0940. Ref AMM 75 725 68.
erts in iblo. Robe92 86. [1,2; A7416] SP

M0938 1,0,2,4,6,12,22,36,62,104,166,268,426,660,1022,1564,2358,3540,5266,
7756,11362,16524,23854,34252,48890,69368,97942,137588,192314,267628
Representation degeneracies for Raymond strings. Ref NUPH B274 544 86. [0,3; A5303]

$$\text{G.f.:}\ \ \Pi\,(1-x^n)^{-c(n)},\ c(n)=0,2,4,3,4,2,4,2,4,2,....$$

M0939 2,4,6,12,24,36,48,60,120,180,240,360,720,840,1260,1680,2520,5040,
7560,10080,15120,20160,25200,27720,45360,50400,55440,83160,110880,166320
Highly composite numbers. Ref RAM1 87. TAMS 56 468 44. HO73 112. Well86 #60. [1,1; A2182,
N0385] Super Abundant Nbrs. $d(A_n) > d(A_{n-1})$ Math. Mag. v64n5 p343-6 Dec 91.

M0940 1,2,4,6,16,12,64,24,36,48,1024,60,4096,192,144,120,65536,180,
262144,240,576,3072,4194304,360,1296,12288,900,960,268435456,720
Smallest number with n divisors. Ref AS1 840. ds. [1,2; A5179]

M0941 1,2,4,6,16,20,24,28,34,46,48,54,56,74,80,82,88,90,106,118,132,140,
142,154,160,164,174,180,194,198,204,210,220,228,238,242,248,254,266,272
n^4+1 is prime. Ref MOC 21 246 67. [1,2; A0068, N0386]

M0942 2,4,6,16,20,36,54,60,96,124,150,252,356,460,612,654,664,698,702,972
17.2^n-1 is prime. Ref MOC 22 421 68. [1,1; A1774, N0387] Reisel 384.

M0943 1,1,2,4,6,19,20,107,116,567,640
Atomic species of degree n. Ref JCT A50 279 89. [1,3; A5227]

f34

Lure of the Integers Joe Roberts A74/6

[2] R. K. Guy, A couple of cubic conundrums, *Amer. Math. Monthly* 91 (1984) 624–29.

2 Minimal Numbers

The least positive integer having exactly h divisors is denoted by $A(h)$. = M0940. Letting $\tau(n)$ be the number of divisors of n we say that n is *minimal* when $A(\tau(n)) = n$; i.e., if n is the least positive integer having the number of divisors it has.

$N!$ is minimal if and only if $1 \leq N \leq 7$.

There exist exactly 14 values of N for which the least common multiple of all integers from 1 to N is minimal and these integers are 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 16, 27, 28.

5040 = 7! is not in the list in M0937.
40320 = 8! , $d(8!) = 96$ but so does $d(27720) = 96$

REFERENCES

M. E. Grost, The smallest number with a given number of divisors, *Amer. Math. Monthly* 75 (1968) 725–29.

3 A Square

"Fermat posed the following question to Wallis: Does there exist a natural number n such that $\sigma(n)$, the sum of the divisors of n, is a perfect square?" Thus starts Halberstam's review of a paper by Schinzel in which the following is proved.

The only prime p such that $\sigma(p^3)$ is a square is $p = 7$.

That is, the only prime number p for which $1 + p + p^2 + p^3$ is a square is the prime number 7. In that case we have $1 + 7 + 7^2 + 7^3 = 20^2$.

See also Integer 3, $\sigma(p^4)$.

In fact, the only positive odd numbers m for which $1 + m + m^2 + m^3$ is a square are 1 and 7. Note that it is not always the case that this sum is equal to $\sigma(m^3)$.

More generally, it appears that in 1877 Gerono showed that the equation

$$1 + x + x^2 + x^3 = y^2$$