

$$A196050(n) = a(n)$$

Complete Prime function reduction

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Prime function $\text{prime}(\cdot) = A000040(\cdot) := \{\cdot\}$ (curly bracket notation).

E.g., $\{4\} = 7$.

Complete prime function reduction $\text{cpfr}(n)$, for $n = 2, \dots, 100$.

E.g., $n = 7$, $7 = \text{prime}(4)$, $4 = 2^2$, $2 = \text{prime}(1)$, and there is no further reduction possible. Hence

$\text{cpfr}(7) = \text{prime}(\text{prime}(1) * \text{prime}(1)) = \text{prime}(\text{prime}(1)^2) = \{\{1\}^2\}$ in curly bracket notation.

$a(n) = A196050(n)$ (Matula-Göbel numbers) equals the number of pairs of curly brackets, i.e. the number of operations of the prime function $\text{prime}(\cdot)$.

E.g., $a(7) = 3$.

$b(n) = A001222(n)$, the number of factors, i.e., the number of prime divisors of n counted with multiplicities.

E.g., $b(7) = 1$.

For the obvious rooted tree interpretation see Figure 2 of the F. Göbel reference [JCT, B29 (1980) 141-143]. There the number of vertices, including the root, are given for $n=1..45$. These numbers are $a(n) + 1$.

n	cpfr(n)	a(n)	b(n)

2	$\{1\}$	1	1
3	$\{\{1\}\}$	2	1
4	$\{1\}^2$	2	2
5	$\{\{\{1\}\}\}$	3	1
6	$\{1\} * \{\{1\}\}$	3	2
7	$\{\{1\}^2\}$	3	1
8	$\{1\}^3$	3	3
9	$\{\{1\}\}^2$	4	2
10	$\{1\} * \{\{\{1\}\}\}$	4	2

11	$\{\{\{\{1\}\}\}\}$	4	1
12	$\{1\}^2 * \{1\}$	4	3
13	$\{\{1\} * \{1\}\}$	4	1
14	$\{1\} * \{1\}^2$	4	2
15	$\{\{1\}\} * \{\{\{1\}\}\}$	5	2
16	$\{1\}^4$	4	4
17	$\{\{\{1\}^2\}\}$	4	1
18	$\{1\} * \{\{1\}\}^2$	5	3
19	$\{\{1\}^3\}$	4	1
20	$\{1\}^2 * \{\{\{1\}\}\}$	5	3
21	$\{\{1\}\} * \{1\}^2$	5	2
22	$\{1\} * \{\{\{\{1\}\}\}\}$	5	2
23	$\{\{\{1\}\}^2\}$	5	1
24	$\{1\}^3 * \{1\}$	5	4
25	$\{\{\{1\}\}\}^2$	6	2
26	$\{1\} * \{\{1\} * \{1\}\}$	5	2
27	$\{\{1\}\}^3$	6	3
28	$\{1\}^2 * \{1\}^2$	5	3
29	$\{\{1\} * \{\{\{1\}\}\}\}$	5	1
30	$\{1\} * \{\{1\}\} * \{\{\{1\}\}\}$	6	3
31	$\{\{\{\{\{1\}\}\}\}\}$	5	1
32	$\{1\}^5$	5	5
33	$\{\{1\}\} * \{\{\{\{1\}\}\}\}$	6	2
34	$\{1\} * \{\{\{1\}^2\}\}$	5	2
35	$\{\{\{1\}\}\} * \{1\}^2$	6	2
36	$\{1\}^2 * \{\{1\}\}^2$	6	4
37	$\{\{1\}^2 * \{1\}\}$	5	1
38	$\{1\} * \{1\}^3$	5	2
39	$\{\{1\}\} * \{\{1\} * \{1\}\}$	6	2
40	$\{1\}^3 * \{\{\{1\}\}\}$	6	4
41	$\{\{\{1\} * \{1\}\}\}$	5	1

42	$\{1\}*\{\{1\}\}*\{\{1\}^2\}$	6	3
43	$\{\{1\}*\{\{1\}^2\}\}$	5	1
44	$\{1\}^2*\{\{\{1\}\}\}$	6	3
45	$\{\{1\}\}^2*\{\{\{1\}\}\}$	7	3
46	$\{1\}*\{\{\{1\}\}^2\}$	6	2
47	$\{\{\{1\}\}*\{\{\{1\}\}\}\}$	6	1
48	$\{1\}^4*\{\{1\}\}$	6	5
49	$\{\{1\}^2\}^2$	6	2
50	$\{1\}*\{\{\{1\}\}\}^2$	7	3
51	$\{\{1\}\}*\{\{\{1\}^2\}\}$	6	2
52	$\{1\}^2*\{\{1\}*\{\{1\}\}\}$	6	3
53	$\{\{1\}^4\}$	5	1
54	$\{1\}*\{\{1\}\}^3$	7	4
55	$\{\{\{1\}\}\}*\{\{\{\{1\}\}\}\}$	7	2
56	$\{1\}^3*\{\{1\}^2\}$	6	4
57	$\{\{1\}\}*\{\{1\}^3\}$	6	2
58	$\{1\}*\{\{1\}*\{\{\{1\}\}\}\}$	6	2
59	$\{\{\{\{1\}^2\}\}\}$	5	1
60	$\{1\}^2*\{\{1\}\}*\{\{\{1\}\}\}$	7	4
61	$\{\{1\}*\{\{1\}\}^2\}$	6	1
62	$\{1\}*\{\{\{\{\{1\}\}\}\}\}$	6	2
63	$\{\{1\}\}^2*\{\{1\}^2\}$	7	3
64	$\{1\}^6$	6	6
65	$\{\{\{1\}\}\}*\{\{1\}*\{\{1\}\}\}$	7	2
66	$\{1\}*\{\{1\}\}*\{\{\{\{1\}\}\}\}$	7	3
67	$\{\{\{1\}^3\}\}$	5	1
68	$\{1\}^2*\{\{\{1\}^2\}\}$	6	3
69	$\{\{1\}\}*\{\{\{1\}\}^2\}$	7	2
70	$\{1\}*\{\{\{1\}\}\}*\{\{1\}^2\}$	7	3
71	$\{\{1\}^2*\{\{\{1\}\}\}\}$	6	1
72	$\{1\}^3*\{\{1\}\}^2$	7	5

73	$\{\{1\}\}*\{1\}^2\}$	6	1
74	$\{1\}*\{1\}^2*\{1\}\}$	6	2
75	$\{\{1\}\}*\{\{\{1\}\}\}^2$	8	3
76	$\{1\}^2*\{1\}^3\}$	6	3
77	$\{\{1\}^2\}*\{\{\{1\}\}\}$	7	2
78	$\{1\}*\{1\}\}*\{1\}*\{1\}\}$	7	3
79	$\{\{1\}*\{\{\{1\}\}\}\}$	6	1
80	$\{1\}^4*\{\{\{1\}\}\}$	7	5
81	$\{\{1\}\}^4$	8	4
82	$\{1\}*\{\{\{1\}*\{1\}\}\}$	6	2
83	$\{\{\{\{1\}\}^2\}\}$	6	1
84	$\{1\}^2*\{1\}\}*\{1\}^2\}$	7	4
85	$\{\{\{1\}\}\}*\{\{\{1\}^2\}\}$	7	2
86	$\{1\}*\{1\}*\{1\}^2\}$	6	2
87	$\{\{1\}\}*\{1\}*\{\{\{1\}\}\}$	7	2
88	$\{1\}^3*\{\{\{\{1\}\}\}\}$	7	4
89	$\{\{1\}^3*\{1\}\}$	6	1
90	$\{1\}*\{1\}\}^2*\{\{\{1\}\}\}$	8	4
91	$\{\{1\}^2\}*\{1\}*\{1\}\}$	7	3
92	$\{1\}^2*\{\{\{1\}\}^2\}$	7	3
93	$\{\{1\}\}*\{\{\{\{\{1\}\}\}\}\}$	7	2
94	$\{1\}*\{\{\{1\}\}*\{\{\{1\}\}\}\}$	7	3
95	$\{\{\{1\}\}\}*\{1\}^3\}$	7	2
96	$\{1\}^5*\{1\}\}$	7	6
97	$\{\{\{\{1\}\}\}^2\}$	7	1
98	$\{1\}*\{1\}^2\}^2$	7	3
99	$\{\{1\}\}^2*\{\{\{\{1\}\}\}\}$	8	3
100	$\{1\}^2*\{\{\{1\}\}\}^2$	8	4

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