

Susanna, 4 to enter

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88-04-08

Neil J.A. Sloane,
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600 Mountain Avenue,
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Dear Neil,

✓ Must be two or three days since I wrote!

✓ This is to enclose a table of Auluck's penny partitions
(ref. is in S.253; I was alerted to it by Jim Propp) and to make
some observations thereon:

✓ An entry, in Auluck's notation, is $R(n,r)$, the number of
arrangements of n pennies in rows, r contiguous pennies in the
bottom row, and each higher row consisting of contiguous pennies,
each touching two pennies in the row below. By contrast, Propp's
penny partitions don't require the second contiguity condition, so
they are somewhat more numerous.

✓ S.253 is the sums of the rows of the table.

✓ I claim that the 19th & 20th entries in Auluck are in error,
and that S.253 should be replaced and extended by

✓ (1), 1, 2, 3, 5, 8, 12, 18, 26, 38, 53, 75, 103, 142, 192, 260, 346,
461, 607 (not 605), 797 (not 796), 1038, 1348, 1738, 2234, 2856, 3638,
4614, 5832, 7342, 9214, 11525, 14369, ... (E. & O.E.)

✓ Sums of *columns* are alternate Fibs. (S.569) but that requires
proof (easy!), I'm copying this to Jim Propp, who, hopefully, will
check. I think he knows most of it already.

✓ A sequence worthy of note is $R(2r,r)$:

✓ 10, 01, 1, 2, 5, 11, 20, 37, 63, 110, 174, 283, 435, 671, 1001
(no array is complete without it), 1492 (a very good year),
2160, 3127, 4442, 6269, 8739, 12109, 16597, 22618, 30576

✓ gms
A1524
Corrected

AS575

gms

SSC

It must be connected with S.542, and the difference between the two sequences,

(..., 0, 0, 0, 0), 1, 2, 10, 19, 52, 105, 224, 429, 820,

may be worth looking at. The particular interest though, is that $R(2r+s, r+s)$, $s = 0, 1, 2, \dots$, is an AP with common difference, $R(r) = \sum R(r, i)$, the members of S.253., which observation simplifies the calculations.

Another sequence which may be of interest, but which is not in Sloane (whenever I say that, I must admit I'm too lazy to look in the Supplement) is

1, 2, 3, 4, 7, 9, 13, 17, 25, 32, 43

It is $R(T_r - s, r)$, $s = 0, 1, 2, \dots, r-1$, extended as far as you want by taking r big enough. Except for its length, the sequence is independent of r ; T_r is the r th triangular number, $\frac{1}{2}r(r+1)$.

Best wishes,

Yours sincerely,

Richard

RKG:1

Richard K. Guy.

P.S. A sequence that you might like, which takes its time at the beginning, is that of the maxima of the rows

1, 1, 2, 3, 4, 5, 6, 7, 9, 11, 15, 20, 27, 35, 44, 56, 73, 91, 115, 148, 186, 227, 283, 358, 435, 538, 671, 813, 1001, 1233, 1492, 1815, 2223, 2673, 3247, 3933, 4713, 5683, 6850, 8170, 9785, 11725, 13948, 16587, 19783, 23468

P.P.S. A nice red herring, swimming between the Capell-Narayana and Wedderburn-Etherington numbers, is the sequence generated by $(1-x-x^2)/(1-2x)(1-x^2)$:

1, 2, 3, 6, 11, 22, 43, 86, 171, 342, 683, 1366, 2731, 5462, 10923, 21846, 43691, 87382, 174763, 349526, 699051, 1398102, 2796203, 5592406, 11184811, 22369622, (compare S.983).

P.P.P.S. Could the Sloane Inter-Library Loan Service (SILLS, as in "sea-lions and sills") provide a photocopy of the relevant portion of R.A. Fisher, Contributions to Mathematical Statistics, Wiley, New York, 1950, to enable me to check S.455? or at least explain

the reference 41.399 (p.399 of what?). NO!!! Forget it!
I found it in MR, which guided me to the *Ann. Eugenics* paper,
excellently reviewed by Coxeter (MR 4,183e).

P.P.P.P.S. (88-04-11) Just heard from you. Will be happy
to include the lost footnote (p.474) in the review of
Conway-Sloane.

encl: 2 sheets

pc: Jim Propp

Nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1	1																															
2		1																														
3			1																													
5				2	1																											
6				1	3	1																										
7					1	2	4	1																								
8					3	3	5	1																								
9					2	5	4	6	1																							
10					1	5	7	5	7	1																						
11						1	5	8	9	6	8	1																				
12							4	10	11	11	7	9	1																			
13							3	11	15	14	13	8	10	1																		
14							2	9	19	20	17	15	9	11	1																	
15							1	9	20	27	25	20	17	10	12	1																
16							1	7	20	32	35	30	23	19	11	13	1															
17							7	20	37	44	43	35	26	21	12	14	1															
18							4	21	38	55	56	51	40	29	23	13	15	1														
19							3	18	44	63	73	68	59	45	32	25	14	16	1													
20	607						2	17	43	73	89	91	80	67	50	35	27	15	17	1												
21	797						1	13	46	79	110	115	109	92	75	55	38	29	16	18	1											
22	1038						1	11	43	89	122	148	141	127	104	83	60	41	31	17	19	1										
23	1348							9	41	93	145	174	186	167	145	116	91	65	44	33	18	20	1									
24	1738							7	37	98	160	209	227	224	193	163	128	99	70	47	35	19	21	1								
25	2234							4	34	97	180	242	283	280	262	219	181	140	107	75	50	37	20	22	1							
26	2856							3	29	98	192	282	333	358	333	300	245	199	152	115	80	53	39	21	23	1						
27	3638							2	24	92	206	317	401	435	433	386	338	271	217	164	123	85	56	41	22	24	1					
28	4614							1	21	89	210	357	465	530	538	508	439	376	297	235	176	131	90	59	43	23	25	1				
29	5832							1	15	83	217	384	543	632	671	641	583	492	414	323	253	188	139	95	62	45	24	26	1			
30	7342							13	75	218	418	609	755	810	813	744	658	545	452	349	271	200	147	100	65	47	25	27	1			
31	9214							9	68	214	442	687	874	988	1001	955	847	733	598	490	375	289	212	155	105	68	49	26	28	1		
32	11525							7	59	212	463	757	1015	1168	1233	1193	1097	950	808	651	528	401	307	224	163	110	71	51	27	29	1	
	14369							4	51	202	478	827	1152	1388	1485	1492	1385	1239	1053	883	704	566	427	325	236	171	115	74	53	28	30	
								3																								
								2																								
								1																								
								1																								

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arrangement of pennies in rows

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arrangement of pennies in rows

	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
30	9	68	214	442	687	874	988	1001	955	847	733	598	490	375	289	212	155	105	68	49	26	28	1				
31	7	59	212	463	757	1015	1168	1233	1193	1097	950	808	651	528	401	307	224	163	110	71	51	27	29	1			
32	4	51	202	478	827	1152	1388	1485	1492	1385	1239	1053	983	704	566	427	325	236	171	115	74	53	28	30	1		
33	3	42	193	490	892	1300	1614	1793	1815	1752	1577	1381	1156	958	757	604	453	343	248	179	120	77	55	29	31	1	
34	2	35	177	495	952	1447	1865	2126	2223	2160	2012	1749	1523	1259	1033	810	642	479	361	260	187	125	80	57	30	32	1
35	1	28	165	492	1006	1600	2129	2505	2673	2667	2506	2272	1961	1665	1362	1108	863	680	505	379	272	195	130	83	59	31	33
36	1	23	146	487	1049	1744	2445	2915	3200	3247	3127	2852	2532	2153	1807	1465	1183	916	718	531	377	284	203	135	86	61	32
37	17	134	470	1031	1889	2706	3755	4784	5933	6836	7588	8198	8792	9245	9549	10019	10505	11005	11518	12045	12587	13145	13718	14307	14901	15500	16104
38	13	116	455	1111	2028	3009	4051	5179	6391	7684	9059	10516	12055	13686	15408	17231	19155	21189	23333	25587	27951	30425	33009	35693	38477	41361	44345
39	9	101	432	1133	2153	3323	4631	6077	7661	9381	11245	13253	15405	17709	20165	22773	25535	28459	31545	34793	38203	41775	45509	49405	53453	57655	62011
40	7	86	407	1132	2276	3630	5199	6981	8985	11219	13677	16359	19175	22135	25249	28517	31939	35515	39245	43129	47167	51359	55705	60205	64859	69669	74635
41	4	73	377	1131	2375	3949	5840	8073	10669	13637	16987	20729	24873	29419	34367	39617	45169	50923	56881	63043	69409	75979	82753	89731	96913	104301	111895
42	3	59	351	1110	2464	4250	6160	8286	10636	13309	16305	19547	23045	26799	30817	35099	39645	44455	49529	54867	60473	66359	72515	78931	85607	92543	99739
43	2	48	316	1092	2530	4552	6796	9379	12405	15885	19729	23947	28549	33535	38907	44565	50509	56739	63255	69957	76845	83919	91171	98601	106311	114301	122571
44	1	39	286	1051	2590	4824	7450	10583	14223	18269	22721	27579	32849	38521	44593	51065	57937	65209	72881	80853	89125	97697	106569	115741	125213	134985	145057
45	1	30	253	1014	2613	5103	8106	11161	13753	16945	20747	25159	30181	35813	42055	48907	56369	64541	73453	83205	93917	105589	118321	132223	147395	163837	181649
46	25	224	962	2634	5327																						
47	17	195																									
48	13	168																									
49	9	142																									
50	7	119																									
51	4	100																									
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