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FURTHER EVIDENCE ON BREAKING TREND FUNCTIONS IN MACROECONOMIC VARIABLES

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data, it is useful to investigate how robust the results are to different postulates. The aim of this paper is to take the extreme view where the choice of the break points is effectively made to be perfectly correlated with the data. This case is instructive to study because if one can still reject the unit root hypothesis under such a scenario it must be the case that it would be rejected under a less stringent assumption.

We proceed as follows for the practical implementation. Again, as in the previous analysis, only one possible break point is allowed for any single series. This break point is first chosen such that the t–statistic for testing the null hypothesis of a unit root is smallest among all possible break points. Hence, using such a procedure, the choice of the break point is indeed perfectly correlated with the data. We also consider choosing the break point that corresponds to a minimal t–statistic on the parameter of the change in the trend function. This allows the mild a priori imposition of a one–sided change (i.e. a decrease in the intercept or the slope of the trend function). As will be seen, such a minor change allows substantial gains in power. We also investigate various issues regarding the choice of the truncation lag parameter in the estimated autoregressions and the effect on the critical values of using different criteria for choosing this lag length.

Our paper is closely related to and complements those of Banerjee, Lumsdaine and Stock (1992) and Zivot and Andrews (1992) in that similar procedures and series are analyzed. We extend their analysis in several directions. On a methodological level, we consider the asymptotic distribution of the sequential test based on the minimal value of the unit root tests over possible break points. We show the results of Zivot and Andrews (1992) to be valid without any trimming at the end points. The proof, which is of interest in itself, is based on projection arguments and introduces a method that can be applied to a variety of frameworks. Concerning the empirical results, our analysis is more extensive and shows that alternative procedures can lead to conclusions that are less favorable to the unit root than suggested in these two studies. We pay particular attention to the importance of the selection of the truncation lag on the outcome of the tests.

The paper is organized as follows. Section 2 reviews the statistical models and statistics involved. Section 3 discusses the asymptotic distribution of the test statistics under the null hypothesis of a unit root. Section 4 analyzes their finite sample distribution using simulation methods. Section 5 contains simulation experiments providing information about their and power under various data–generating processes. Section 6 presents the empirical results for the Nelson–Plosser (1982) data set. Section 7 analyzes an international

In the case where the noise function is assumed to be generated from a finite order autoregressive process, we can use results in Hall (1990) to show that the data–dependent methods described above lead to tests having the same asymptotic properties as would prevail if the true autoregressive order was selected to estimate the autoregression provided kmax is selected greater than the true value. In the more general case where moving–average components are permitted, Ng and Perron (1994) show that tests with such data dependent methods to select k have the same asymptotic distribution provided $kmax^3/T$ converges to 0.

We choose these "general to specific" procedures rather than methods based on information criteria, such as AIC, because the latter tend to select very parsimonious models leading to tests with sometimes serious size distortions and/or power losses. This finite sample performance is consistent with the finding of Ng and Perron (1994) who show that using an information criterion leads to a selected value of k that increases to infinity, as T increases, only at the very slow rate log(T). These theoretical results are in accord with various empirical results showing that using the AIC leads to very small values of k being selected (typically 0 or 1) and that oftentimes the estimated residuals exhibit serial correlation (see Perron (1994)).

3. THE ASYMPTOTIC DISTRIBUTION OF THE STATISTICS.

In this section, we consider the limiting distribution of the statistics. To simplify the derivations we suppose the data–generating process to be a random walk,

$$y_t = y_{t-1} + e_t, \qquad (t = 0, 1, ..., T) \tag{4}$$

where the errors e_t are martingale differences, and consider the statistics constructed with $k = 0$. Using arguments in Ng and Perron (1994), we can then state that the resulting limiting distribution remains the same when additional correlation is present and the statistics are constructed with one of the data–dependent method to select k. This holds provided $kmax^3/T \to 0$ as $T \to \infty$. This is the same strategy as used by Zivot and Andrews (1992) and Banerjee, Lumsdaine and Stock (1992). All statistics are asymptotically invariant to a change in intercept. Vogelsang and Perron (1994) show that they are not asymptotically invariant to a change in slope but that the asymptotic distribution corresponding to a zero change in slope is a better approximation to the finite sample distribution for values typically encountered in practice. The following Theorem concerning

These theoretical issues are consistent with the empirical results of BLS who report values of k at 0 or 1 for all countries when using an information criterion. In no cases does our methods select such low values (except for Italy where we both agree for a non–rejection).

8. EMPIRICAL RESULTS FOR SOME ADDITIONAL SERIES.

Some additional series from alternative sources are analyzed in this Section. First, for the Real per capita GNP and Money Supply series, we use data sources other than the Nelson–Plosser data set. As discussed in Section 6, rejections of the unit root are borderline for these series when allowance is made for an unknown break point without imposing a one–sided change. To provide alternative evidence, we first present in Table 11 results related to the Friedman and Schwartz (1982) Real per capita GNP series for the same period (1909–1970), which is graphed in Figure 8. The results imply a maximum p–value of .03 under any method to select k and T_b [5], allowing an easy rejection of the unit root hypothesis for this series.

Consider now an alternative source for the money supply variable, the annual M2 series supplied in Balke and Gordon (1986) from 1869 to 1973, graphed in Figure 9. The results in Table 11 again show a strong rejection of the unit root with a p–value of at most .05 under any procedure.

Following the work of Hall (1978), much interest has been given to the time series behavior of consumption. To this effect, we analyze a data set consisting of historical series covering 1889 to 1973 for Nominal Consumption, Real Consumption, their per capita counterparts, the Consumption Price Index and also the Population series. These data are a subset of those used in Grossman and Shiller (1981). The graph of these series are presented in Figures 10 through 15. The results concerning the unit root tests are also presented in Table 11. For the Nominal and Real Consumption series the unit root can be rejected with a p–value less than .01 under any procedure. The series again exhibit a significant decline in their level in 1929. For the Nominal per capita Consumption series, a rejection is still possible with p–values at most .03 but the picture is different with the

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5 The rejection of the unit root for the Friedman and Schwartz series is robust to using the longer samples 1900–1973 and 1890–1973. It is not robust to using the whole sample 1869–1973. In the latter case, however, the unit root can be rejected using a standard Dickey–Fuller procedure without any allowance for a possible change in the trend function.

APPENDIX: Proof of Theorem 1.

To simplify cross–references, we adopt the notation of Zivot and Andrews (1992), henceforth referred to as Z–A. Let $S_t = \Sigma_{j=1}^{t} e_j$ ($S_0 = 0$) and $X_T(r) = \sigma^{-1}T^{-1/2}S_{[Tr]}$, $(j-1)/T \leq r < j/T$ (for $j = 1, ..., T$), where $\sigma^2 = \lim_{T\to\infty} T^{-1}E(S_T^2)$ and $[\cdot]$ denotes the integer part of the argument. Since $\{e_t\}$ is i.i.d. with finite variance, we have $X_T(r) \Rightarrow W(r)$, where $\Rightarrow$ denotes weak convergence in distribution (from the space $D[0,1]$ to the space $C[0,1]$ using the uniform metric on the space of functions on $[0,1]$) with $W(r)$ a standard Wiener process on $[0,1]$. Also, $\sigma_T^2 \equiv T^{-1}\Sigma_1^T e_t^2 \to_p \sigma^2$ where $\to_p$ denotes convergence in probability. Omitting the one–time dummy variable $D(T_b)_t$ (since it is asymptotically negligeable), we consider the following regressions:

$$y_t = \beta^i(\lambda)\, z_{tT}^i(\lambda) + \alpha^i(\lambda)\, y_{t-1} + e_t\,, \qquad (t = 1, ..., T), \tag{A.1}$$

for models $i = 1, 2$. The vector $z_{tT}^i(\lambda)$ encompasses the deterministic components of the model and depends explicitly on λ, the break fraction, and T, the sample size. For example, $z_{t,T}^1(\lambda)' = (1, t, DU_t(\lambda))$. Let $Z_T^i(\lambda,r) = \delta_T^i\, z_{[Tr],T}^i(\lambda)$ be a rescaled version with δ_T^i a diagonal matrix of weights. For example, $\delta_T^1 = \text{diag}(1, T^{-1}, 1)$. We also define the limiting functions $Z^1(\lambda,r) = (1, r, du(\lambda,r))'$ where $du(\lambda,r) = 1(r > \lambda)$, and $Z^2(\lambda,r) = (1, r, du(\lambda,r), dt^*(\lambda,r))'$ where $dt^*(\lambda,r) = 1(r > \lambda)(r - \lambda)$. Note that, as argued in Z–A, we do not have $Z_T^i(\lambda,r) \Rightarrow Z^i(\lambda,r)$ (i = A, C) as $T \to \infty$, using the uniform metric on the space of functions on $D[0, 1]$. The proof nevertheless remains valid without the need to introduce another metric to guarantee such convergence results. For simplicity, we henceforth drop the subscript denoting the model.

It is convenient to first transform (A.1) as follows. Let $Pz_T(\lambda) = [Pz_{1,T}(\lambda), ..., Pz_{T,T}(\lambda)]$ be the linear map projecting onto the space spanned by the columns of $z_T(\lambda)' = (z_{1,T}(\lambda), ..., z_{T,T}(\lambda))$. By definition $Pz_T(\lambda) = z_T(\lambda)(z_T'(\lambda)z_T(\lambda))^{-}z_T(\lambda)'$ where $(\cdot)^{-}$ denotes a g–inverse. Premultiplying by $Mz_T(\lambda) \equiv (I - Pz_T(\lambda))$, (A.1) can be written, in matrix notation, as:

$$Mz_T(\lambda)Y = \alpha(\lambda)Mz_T(\lambda)Y_{-1} + Mz_T(\lambda)e\,, \tag{A.2}$$

$\inf_{\lambda\epsilon[0,1]} t_{\hat{\alpha}}(\lambda) =$

$$g(X_T(r), \int_0^1 X_T(r)dX_T(r), Pz_T(\lambda)X_T(r), \int_0^1 Pz_T(\lambda)X_T(r)dX_T(r), s_T(\lambda)) + o_{p\lambda}(1),$$

where

$$g = h^*[h[H_1[X_T(r), Pz_T(\lambda)X_T(r)], H_2[\int_0^1 X_T(r)dX_T(r), \int_0^1 Pz_T(\lambda)X_T(r)dX_T(r)], s_T(\lambda)]],$$

with $h^*(m) = \inf_{\lambda\epsilon[0,1]} m(\lambda)$ for any real function $m = m(\cdot)$ on [0, 1]; and for any real functions $m_1(\cdot)$, $m_2(\cdot)$, $m_3(\cdot)$ on [0,1], $h[m_1(\lambda), m_2(\lambda), m_3(\lambda)] = m_1(\lambda)^{-1/2} m_2(\lambda)/m_3(\lambda)$. The functional H_1 and H_2 are defined by (A.3) and (A.4). The weak convergence results for each of the elements is contained in the following lemma.

Lemma A.1: The following convergence results hold jointly:

a) $X_T(r) \Rightarrow W(r)$;

b) $\int_0^1 X_T(r)dX_T(r) \Rightarrow \int_0^1 W(r)dW(r)$;

c) $Pz_T(\lambda)X_T(r) \Rightarrow Pz(\lambda)W(r) \equiv Z(\lambda,r)'[\int_0^1 Z(\lambda,s)Z(\lambda,s)'ds]^- \int_0^1 Z(\lambda,s)W(s)ds$;

d) $\int_0^1 P_T z(\lambda)X_T(r)dX_T(r) \Rightarrow \int_0^1 Pz(\lambda)W(r)dW(r)$;

e) $s_T^2(\lambda) = \sigma^2 + o_{p\lambda}(1)$.

Parts (a) and (b) are standard results, and part (e) follows using (c) and (d) and the fact that $T^{-1}\Sigma_1^T e_t \rightarrow_p \sigma^2$. To prove part (c), we start with the following Lemma which follows from Theorem 5.5 of Billingsley (1968).

Lemma A.2: $Pz_T(\lambda)X_T(r) \Rightarrow Pz(\lambda)W(r)$ if $X_T(r) \Rightarrow W(r)$ and for any sequence of functions $\{v_T(s)\}$ $(0 \leq s \leq 1)$ approaching $v(s)$, we have:

$$Pz_T(v_T(s)) \rightarrow Pz(v(s)), \tag{A.5}$$

where $Pz_T(v_T(s)) = Z_T(\lambda,r)'[\int_0^1 Z_T(\lambda,s)Z_T(\lambda,s)'ds]^- \int_0^1 Z_T(\lambda,s)v_T(s)ds$.

and $Pz(v(s)) = Z(\lambda,r)'[\int_0^1 Z(\lambda,s)Z(\lambda,s)'ds]^- \int_0^1 Z(\lambda,s)v(s)ds$.

Proof: Since H_1 and H_2 are continuous functions of their respective elements, the proof follows if each of the elements is bounded over [0,1] with W–probability one. $W(\cdot)$ is bounded with W–probability one and so is $\int_0^1 W(r)dW(r)$ as discussed in Z–A. Using arguments similar to those in Z–A, $\int_0^1 Pz(\lambda)W(r)dW(r)$ will be continuous if $Pz(\lambda)W(r)$ is continuous, i.e. if $\sup_{\lambda\epsilon[0,1]}|Pz(\lambda)W(r)| < \infty$. We note that $Pz(\cdot)$ is a linear operator that maps an element on C[0, 1] (the Wiener process W(r) which is continuous) to a subspace defined by the functions $Z(\lambda,r)$. Continuity of $Pz(\lambda)W(r)$ follows since a linear projection map is bounded and continuous (see, e.g., Ash (1972), p. 130 and p. 148). □

It is useful to illustrate this result by way of an example. Consider Model A where $Z(\lambda, r) = (1, r, du(\lambda, r))$. Note that :

$$\int_0^1 Z(\lambda,s)Z(\lambda,s)'ds = \begin{bmatrix} 1 & 1/2 & (1-\lambda) \\ 1/2 & 1/3 & (1-\lambda^2)/2 \\ (1-\lambda) & (1-\lambda^2)/2 & (1-\lambda) \end{bmatrix}.$$

If $\lambda = 0$, $\int_0^1 Z(0,s)Z(0,s)'ds = \begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1/3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \equiv A$,

and if $\lambda = 1$, $\int_0^1 Z(1,s)Z(1,s)'ds = \begin{bmatrix} 1 & 1/2 & 1 \\ 1/2 & 1/3 & 1/2 \\ 1 & 1/2 & 1 \end{bmatrix} \equiv B$.

A and B are obviously nonsingular, but a common g–inverse is given by

$$G = 12 \begin{bmatrix} 1/3 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Since the choice of the g–inverse leaves a projection map unchanged, we have for $\lambda = 0, 1$:

$$Pz(\lambda)W(r) = Z^{\perp}(r)'[\int_0^1 Z^{\perp}(s)Z^{\perp}(s)'ds]^{-1}\int_0^1 Z^{\perp}(s)W(s)ds ,$$

where $Z^{\perp}(r)' = (1, r)$, in which case the limiting distribution of $t_{\hat{\alpha}}(\lambda)$ ($\lambda = 0, 1$) reduces to that in the case where no dummy for structural change is included.

The proof for Model 3 follows similar arguments and is therefore omitted. It uses the limiting distribution for fixed λ derived in Perron and Vogelsang (1993a,b) (see also Vogelsang (1993)).

TABLE 4: Finite Sample Size and Power Simulations; Model 3, $t_{\alpha}^{*}(3)$.

DGP: $y_t = \gamma DT_t^* + \tilde{y}_t \; ; \; \tilde{y}_t = \alpha\tilde{y}_{t-1} + \Sigma_{i=1}^{4}\phi(i)\Delta\tilde{y}_{t-i} + (1 + \psi L)e_t$

$e_t \sim$ i.i.d. $N(0, 1)$; $T = 100$, $T_b = 50$; 2,000 replications; 5% nominal size; kmax = 5.

	Size ($\alpha = 1.0$)					Power ($\alpha = 0.8$)				
	γ					γ				
k	0.0	0.1	0.3	0.5	1.0	0.0	0.1	0.3	0.5	1.0
(1) $\phi(i) = 0.0$ (i=1,..,4), $\psi = 0.0$										
0	.049	.053	.055	.047	.036	.358	.365	.344	.331	.325
1	.044	.049	.048	.042	.037	.287	.299	.283	.277	.27[illegible]
2	.045	.046	.048	.041	.040	.203	.215	.207	.199	.205
3	.038	.039	.042	.040	.047	.160	.177	.169	.165	.167
4	.035	.037	.039	.036	.041	.122	.129	.134	.130	.134
5	.035	.035	.039	.038	.039	.110	.123	.125	.116	.117
F–sig	.050	.054	.058	.055	.050	.235	.256	.244	.231	.233
t–sig	.049	.051	.058	.050	.045	.257	.278	.270	.259	.258
(2) $\phi(1) = 0.6$, $\psi = \phi(i) = 0.0$ (i=2,3,4)										
0	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000
1	.058	.060	.056	.054	.062	.908	.903	.904	.902	.901
2	.046	.048	.049	.049	.055	.753	.758	.761	.753	.756
3	.045	.046	.040	.041	.045	.586	.592	.600	.594	.593
4	.037	.040	.034	.038	.044	.405	.426	.424	.417	.417
5	.033	.033	.034	.037	.045	.289	.302	.306	.305	.305
F–sig	.049	.051	.047	.047	.054	.676	.679	.679	.688	.693
t–sig	.049	.047	.038	.046	.049	.760	.773	.774	.778	.785
(3) $\phi(1) = -0.6$, $\psi = \phi(i) = 0.0$ (i=2,3,4)										
0	.858	.874	.873	.858	.848	.997	.997	.998	.998	.998
1	.051	.048	.043	.038	.034	.131	.132	.117	.114	.113
2	.046	.040	.040	.037	.034	.090	.100	.096	.094	.098
3	.044	.045	.041	.037	.039	.084	.098	.094	.099	.099
4	.034	.030	.033	.032	.033	.063	.074	.082	.083	.077
5	.033	.035	.038	.037	.039	.056	.073	.073	.075	.070
F–sig	.037	.040	.044	.042	.044	.091	.104	.104	.104	.100
t–sig	.039	.039	.042	.034	.037	.090	.105	.105	.104	.097
(4) $\phi(1) = 0.4$, $\phi(2) = 0.2$, $\psi = \phi(3) = \phi(4) = 0.0$										
0	.004	.004	.005	.004	.004	.001	.000	.000	.000	.000
1	.009	.008	.008	.007	.006	.432	.439	.428	.421	.424
2	.048	.051	.050	.049	.048	.756	.764	.765	.763	.760
3	.040	.042	.047	.044	.051	.598	.611	.611	.607	.602
4	.038	.039	.043	.044	.046	.413	.432	.438	.436	.421
5	.042	.043	.040	.040	.050	.300	.314	.318	.311	.313
F–sig	.040	.048	.049	.047	.050	.582	.593	.600	.591	.593
t–sig	.038	.040	.038	.040	.044	.607	.625	.626	.624	.620

TABLE 5: Finite Sample Size and Power Simulations; Model 3, BLS method.
DGP: $y_t = \gamma DT_t^* + \tilde{y}_t$; $\tilde{y}_t = \alpha\tilde{y}_{t-1} + \Sigma_{i=1}^4 \phi(i)\Delta\tilde{y}_{t-i} + (1 + \psi L)e_t$
$e_t \sim$ i.i.d. N(0, 1) ; T = 100, T_b = 50 ; 2,000 replications; 5% nominal size; k = 4.

	Size ($\alpha = 1.0$)					Power ($\alpha = 0.8$)				
	γ					γ				
	0.0	0.1	0.3	0.5	1.0	0.0	0.1	0.3	0.5	1.0
(1) $\phi(i) = 0.0$ (i=1,..,4), $\psi = 0.0$										
	.073	.071	.089	.131	.320	.149	.173	.205	.292	.706
(2) $\phi(1) = 0.6$, $\psi = \phi(i) = 0.0$ (i=2,3,4)										
	.091	.090	.080	.091	.126	.452	.468	.479	.494	.650
(3) $\phi(1) = -0.6$, $\psi = \phi(i) = 0.0$ (i=2,3,4)										
	.067	.068	.093	.190	.660	.089	.124	.174	.347	.899
(4) $\phi(1) = 0.4$, $\phi(2) = 0.2$, $\psi = \phi(3) = \phi(4) = 0.0$										
	.090	.089	.082	.094	.130	.445	.470	.484	.501	.650
(5) $\phi(1) = .3$, $\phi(2) = .3$, $\phi(3) = .25$, $\phi(4) = .14$, $\psi = 0.0$										
	.168	.173	.166	.169	.172	.910	.917	.919	.923	.922
(6) $\psi = 0.5$, $\phi(i) = 0.0$ (i=1,..,4)										
	.068	.067	.074	.087	.165	.140	.146	.178	.188	.421
(7) $\psi = -0.4$, $\phi(i) = 0.0$ (i=1,..,4)										
	.072	.075	.111	.208	.690	.208	.239	.309	.551	.974

TABLE 6: Empirical Results, Nelson – Plosser Data; $t^*_{\alpha}(1)$, kmax = 10.

Regression: $y_t = \mu + \theta DU_t + \beta t + \delta D(T_b)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_b	k	$t_{\hat{\theta}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (asy)	p–value (F–sig)	p–value (t–sig)
Real GNP	1909–1970	62	1928	9	–5.13	.190	–5.93	<.01	<.01	
			1928	8	–4.79	.267	–5.50	<.01		.03
Nominal GNP [a]	1909–1970	62	1928	11	–6.34	.404	–8.16	<.01		<.01
			1928	15	–5.94	.497	–6.21	<.01	<.01	
Real per Capita GNP	1909–1970	62	1928	9	–3.73	.313	–4.81	.06	.12	
			1928	7	–3.31	.484	–4.51	.13		.21
Industrial Production	1860–1970	111	1928	8	–5.18	.272	–6.01	<.01	<.01	<.01
Employment	1890–1970	81	1928	8	–3.42	.586	–5.14	.02	.05	
			1928	7	–3.11	.650	–4.91	.04		.09
GNP Deflator	1889–1970	82	1928	5	–3.28	.783	–4.14	.29	.35	.35
C.P.I.	1860–1970	111	1939	5	2.00	.948	–3.09	.88	.88	.88
Wages	1900–1970	71	1929	7	–4.32	.619	–5.41	<.01		.02
			1929	9	–4.10	.635	–4.62	.10	.16	
Money Stock	1889–1970	82	1929	7	–2.80	.783	–4.69	.08	.14	
			1927	6	–2.50	.831	–4.30	.21		.28
Velocity	1869–1970	102	1949	8	2.95	.830	–2.81	.95	.94	
			1946	0	3.24	.858	–3.29	.81		.81
Interest Rate	1900–1970	71	1965	3	3.86	.934	–1.35	>.99	>.99	
			1963	3	3.44	.928	–1.35	>.99		>.99

a : For Nominal GNP, kmax = 15 (See footnote 3).

Table 7: Empirical Results; Nelson–Plosser Data Set; Model 1.

$t^*_{\hat{\alpha},\hat{\theta}}(1)$; Choosing T_b minimizing $t_{\hat{\theta}}$; kmax = 10.

Regression: $y_t = \mu + \theta DU_t + \beta t + \delta D(T_b)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	T_b	k	$t_{\hat{\theta}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (asy)	p–value (F–sig)	p–value (t–sig)
Real GNP	1928	9	–5.13	.190	–5.93	<.01	<.01	
	1928	8	–4.79	.267	–5.50	<.01		.02
Nominal GNP[a]	1929	11	–6.78	.231	–7.86	<.01		<.01
	1928	15	–5.94	.497	–6.21	<.01	<.01	
Real Per Capita GNP	1928	9	–3.73	.313	–4.81	.03	.06	
	1928	7	–3.31	.484	–4.51	.07		.10
Industrial Production	1928	8	–5.18	.272	–6.01	<.01	<.01	<.01
Employment	1928	8	–3.42	.586	–5.14	.01	.02	
	1928	7	–3.11	.650	–4.91	.02		.04
GNP Deflator	1919	5	–3.51	.886	–3.24	.58		.54
	1919	9	–3.61	.829	–3.87	.27	.28	
C.P.I.	1919	5	–3.12	.982	–1.16	.98	.96	.96
Wages	1929	7	–4.32	.619	–5.41	<.01		.01
	1929	9	–4.10	.635	–4.62	.05	.08	
Money Stock	1929	7	–2.80	.783	–4.69	.04	.07	
	1928	6	–2.63	.824	–4.28	.12		.15
Velocity	1880	5	–2.74	.928	–1.62	.96	.93	
	1880	0	–2.46	.897	–2.43	.87		.83
Interest Rate	1920	0	–4.16	1.058	1.16	>.99		>.99
	1918	0	–3.59	1.079	2.08	>.99	>.99	

[a] : For Nominal GNP, kmax = 15 (see footnote 3).

TABLE 11: Empirical Results, Additional Series; $t^*_\alpha(1)$, kmax = 12.

Regression: $y_t = \mu + \theta DU_t + \beta t + \delta D(T_b)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_b	k	$t_{\hat{\theta}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (asy)	p–value (F–sig)	p–value (t–sig)
Real per Capita GNP (FS)	1909–1970	62	1928	11	–4.74	.202	–5.42	<.01	.03	.03
M2	1869–1973	105	1929	12	–4.21	.720	–4.69	.08	.13	.14
Nominal Consumption	1889–1973	85	1928	11	–6.00	.579	–6.78	<.01		<.01
			1928	12	–5.65	.614	–5.70	<.01	.01	
Real Consumption	1889–1973	85	1928	11	–5.96	.202	–6.45	<.01	<.01	
			1929	11	–5.78	.109	–6.19	<.01		<.01
Nominal Per Capita Cons.	1889–1973	85	1928	12	–5.12	.613	–5.25	.02	.03	.03
Real Per Capita Consumption	1889–1973	85	1928	12	–4.14	.174	–4.49	.13	.20	
			1928	10	–3.69	.371	–4.54	.12		.19
Consumption Price Index	1889–1973	85	1929	8	–3.77	.709	–4.71	.07	.13	
			1919	10	–3.86	.810	–4.34	.19		.26
Population	1889–1973	85	1917	11	3.35	.933	–4.82	.05	.10	
			1923	10	–1.94	.948	–3.48	.70		.71

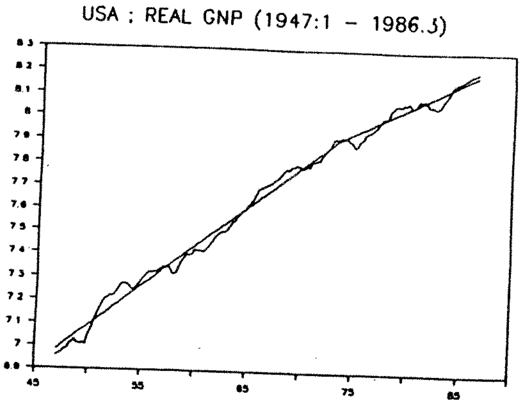


Figure 1

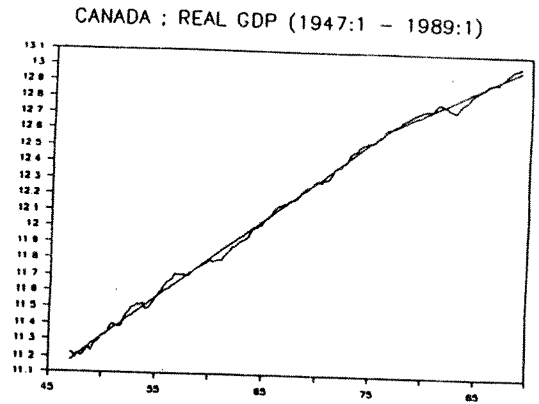


Figure 2

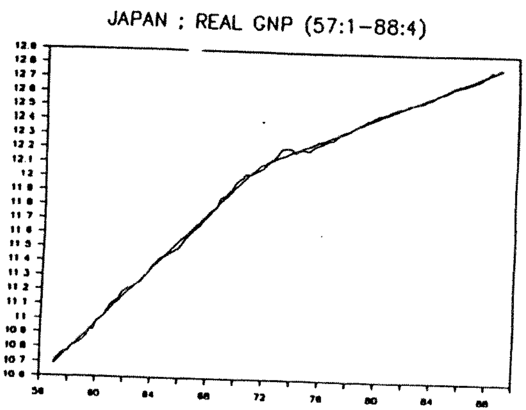
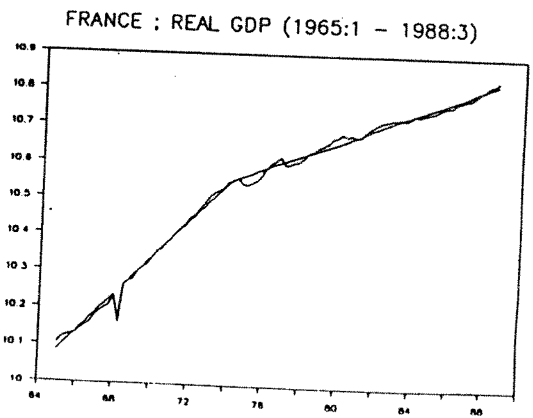


Figure 3



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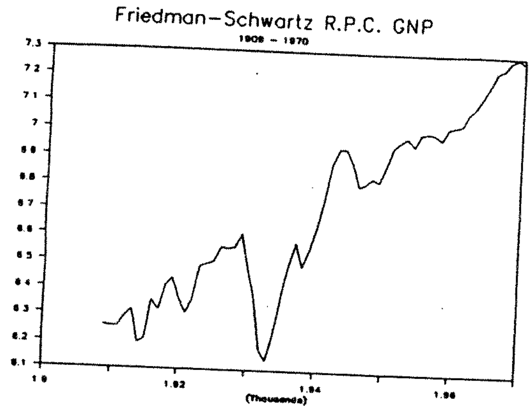

Figure 8

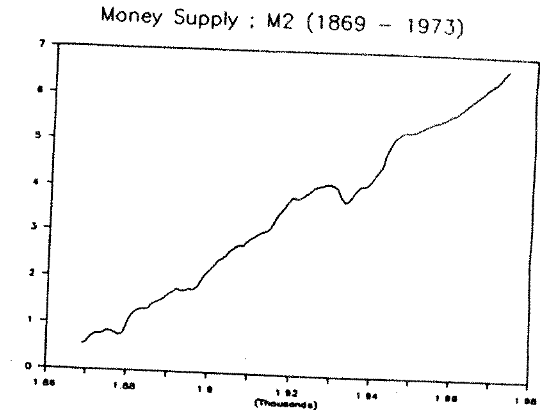

Figure 9

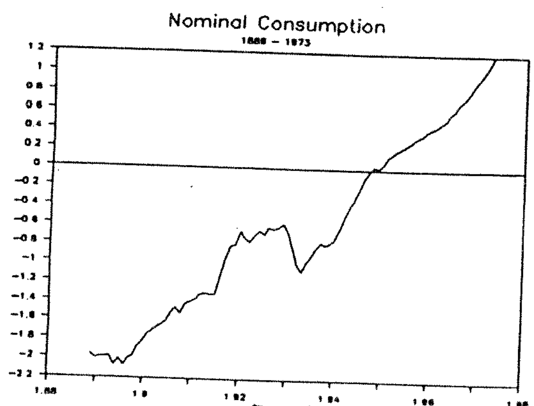

Figure 10

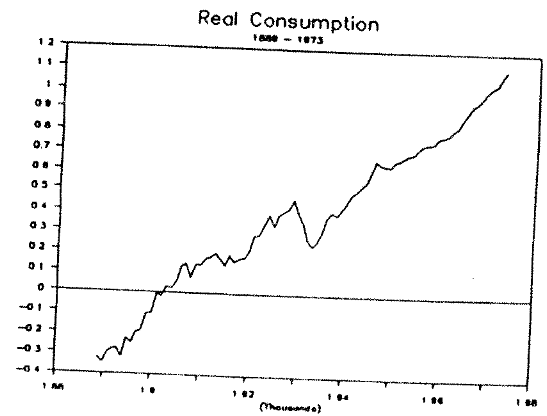

Figure 11

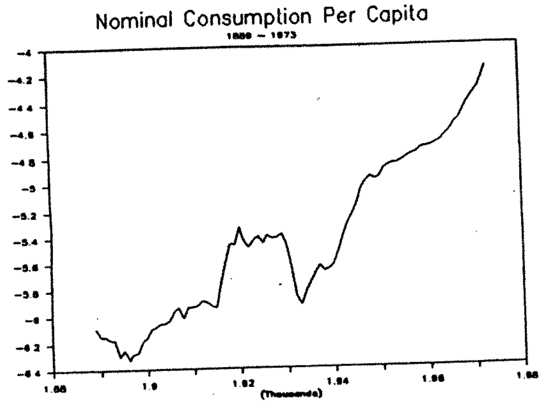

Figure 12

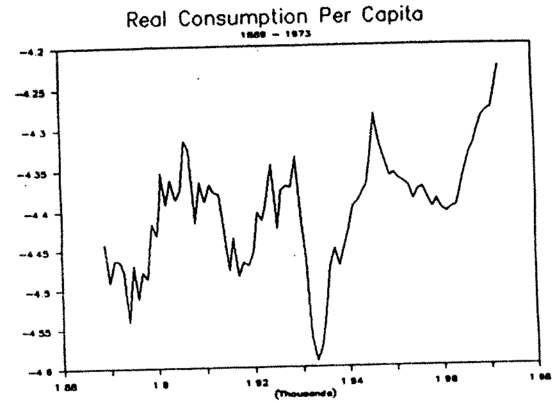

Figure 13

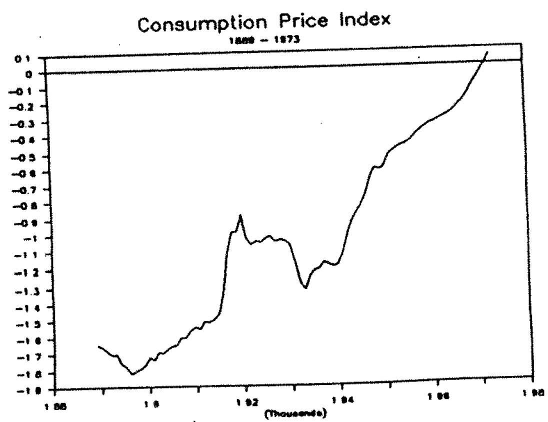

Figure 14

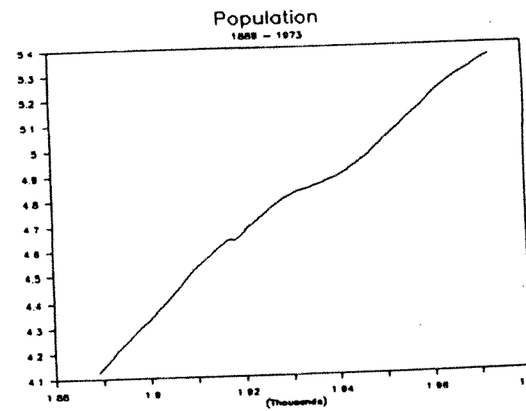

Figure 15

9416 : Ghysels, Eric et Haldun Sarlan, "On the Analysis of Business Cycles Through the Spectrum of Chronologies", août 1994, 37 pages.

9417 : Martel, Jocelyn et Timothy C.G. Fisher, "The Creditors' Financial Reorganization Decision : New Evidence from Canadian Data", août 1994, 21 pages.

9418 : Cannings, Kathy, Claude Montmarquette et Sophie Mahseredjian, "Entrance Quotas and Admission to Medical Schools : A Sequential Probit Model", septembre 1994, 26 pages.

9419 : Cannings, Kathy, Claude Montmarquette et Sophie Mahseredjian, "Major Choices : Undergraduate Concentrations and the Probability of Graduation", septembre 1994, 26 pages.

9420 : Nabeya, Seiji et Pierre Perron, "Approximations to Some Exact Distributions in the First Order Autoregressive Model with Dependent Errors", septembre 1994, 40 pages.

9421 : Perron, Pierre, "Further Evidence on Breaking Trend Functions in Macroeconomic Variables", octobre 1994, 50 pages.