$Deff$ $Deff$ D^2_{eft} $Deff$

$Deff$ $Deff$

$Deff > 1$ $Deff < 1$

$Deff = 1$

$Deff < 1$

$Deff > 1$

$var(\hat{\theta}_w)$	$\hat{\theta}_w$
$var(\hat{\theta}_{SRSWOR})$	$\hat{\theta}_{SRSWOR}$
$var(\hat{\theta}_{SRSWR})$	$\hat{\theta}_{SRSWR}$
$Deff$ $Deff$ $$Deff_p(\hat{\theta}) = \frac{var(\hat{\theta}_w)}{var(\hat{\theta}_{SRSWOR})}$$	
$Deft$ $Deft$ $$Deft = \sqrt{\frac{var(\hat{\theta}_w)}{var(\hat{\theta}_{SRSWR})}}$$	
n	
N	
n_{eff} $$n_{\text{eff}} = \frac{n}{Deff}$$	
w_i	

n_h
N_h
w_h
H
$n^* \bar{b}$
K
n_k
ρ
$L \, C_V^2 \; relvar(w)$
$\hat{\rho}_{y_i P}$
$\hat{\alpha}$
$\hat{\sigma}_y$
$cv_w \, cv_P$
f
S_y^2
$p_i \; P_i$
π_i

$Deff$ D_{eff}

θ

$\hat{\theta}_{SRSWOR}$

$\hat{\theta}_w$

p

$$Deff_p(\hat{\theta}) = \frac{var(\hat{\theta}_w)}{var(\hat{\theta}_{SRSWOR})}$$

$Deff$

$Deff$

$Deff$

$$Deff_p = \frac{var_p(\bar{y}_p)}{(1-f)S_y^2/n}$$

n $f = n/N$

$(1-f)$

S_y^2

$var_p(\bar{y}_p)$

$Deff$

$Deff$

$Deff$

$Deft$ $Deft$

$$Deft = \sqrt{\frac{var(\hat{\theta}_w)}{var(\hat{\theta}_{SRSWR})}}$$

$Deft^2$ $Deff$

$Deft$

$\sqrt{Deff}$

$Deft$

$Deff$ $Deft \approx \sqrt{Deff}$

$Deft$

$$n_{\text{eff}} = \frac{n}{Deff}$$

$Deff$

$$\frac{n_{eff}}{n} = \frac{1}{Deff}$$

$$Deff = \frac{n\sum_{i=1}^{n} w_i^2}{(\sum_{i=1}^{n} w_i)^2} = \frac{\overline{w^2}}{\overline{w}^2}$$

w_i i

$$Deff = 1 + (n^* - 1)\rho$$

n^* ρ

$$Deff = \frac{n\sum_{h=1}^{H}(n_h w_h^2)}{(\sum_{h=1}^{H} n_h w_h)^2}(1 + (n^* - 1)\rho)$$

n_h w_h h

$$Deff = (1 - \hat{\rho}_{y,P}^2)(1 + L) + (\frac{\hat{\alpha}}{\hat{\sigma}_y})^2 L$$

$\hat{\rho}_{y,P}$ L $\hat{\alpha}$ $\hat{\sigma}_y$

$$Deff = (1 - \hat{\rho}_{y,P}^2)(1 + cv_w^2) + \frac{\hat{\rho}_{y,P}^2}{cv_P^2} cv_w^2$$

cv_w cv_P

Unequal selection probabilities

Sources of unequal selection probabilities

h

$f_h \propto \frac{S_h}{\sqrt{C_h}}$

S_h

h C_h

h

$$n_h = n \frac{W_h S_{Uh}}{\sum_h W_h S_{Uh}}$$

n_h

$W_h = \frac{N_h}{N}$

S_{Uh}

p_h

h

$$\frac{N}{n} = \frac{1}{f}$$

n

N

$$w_i = \frac{1}{p_i}$$

$\hat{\hat{Y}}$

n H

$\sum_{h=1}^{H} n_h = n$

n_h

h w_h w_h

h

$$n \sum^{H} (n_h w_h^2)$$

$\forall h : n_h = 1$

$$Deff = \frac{n \sum_{i=1}^{n} w_i^2}{\left(\sum_{i=1}^{n} w_i\right)^2} = \frac{\frac{1}{n} \sum_{i=1}^{n} w_i^2}{\left(\frac{1}{n} \sum_{i=1}^{n} w_i\right)^2} = \frac{\overline{w^2}}{\overline{w}^2}$$

$$n_{\text{eff}} = \frac{\left(\sum_{i=1}^{n} w_i\right)^2}{\sum_{i=1}^{n} w_i^2}$$

$$Deff_{kish} = \frac{var(\bar{y}_w)}{var(\bar{y}')} = \frac{var\left(\frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i}\right)}{var\left(\frac{\sum_{i=1}^{n} y'_i}{n}\right)}$$

$\forall(i \neq j) : cor(y_i, y_j) = 0$ $y_1, \ldots, y_n$ σ^2

$y = y'$ $var(\bar{y}_w)$ $\overline{var(\bar{y}_w)} = \overline{var(\bar{y})} \times Deff$

y_i

y_i

$$Deff = 1 + L = 1 + {C_V}^2 = 1 + relvar(w) = 1 + \frac{V(w)}{\bar{w}^2}$$

$V(w) = \frac{\sum(w_i - \bar{w})^2}{n}$ w $\bar{w} = \frac{\sum w_i}{n}$

${C_V}^2 = V(w)$

$Deff = 1 + V(w)$

$Deff$

$Deff_{kish}$ $Deft^2_{kish}$ $deff_K$

$\hat{Y}$

y_k P_i $\sum_{i=1}^{N} P_i = 1$

p_k

$\sum_U p_k = 1$

$w_i = \frac{1}{nP_i}$

y_k p_k

$Z_i = \frac{y_k}{p_k}$

$E[w_i] = 1$

$E[Z_i] = E[I_i \frac{y_k}{p_k}] = \frac{y_k}{p_k} E[I_i] = \frac{y_k}{p_k} p_k = y_k$

y_i P_i

$\hat{Y}_{pwr} = \frac{1}{n} \sum_i^n Z_i$

$y_i = \alpha + \beta P_i + \epsilon_i$

$\rho_{y,P}$ $\rho_{y,P}$

$cov(y_i, y_j) = 0$

$var(y_i) = \sigma_h^2 = \sigma^2$

n^*

n_k $Deff = 1 + (n^* - 1)\rho.$

$n = \sum n_k$ $Deff_C$

n^*

$\bar{b}$

ρ

i

j k

$cov(y_i, y_j) = \rho\sigma^2$

$\times$

$$Deff_{Kish} = \frac{n \sum_{h=1}^{H} (n_h w_h^2)}{\left(\sum_{h=1}^{H} n_h w_h\right)^2} (1 + (n^* - 1)\rho) = deff_k \times deff_C$$

$\times$

$\times$

$Deft$

$Deft$

$\frac{S_n^2}{n}$

$Deft$

y_i p_i

$Deft$

S_i i

$P(\text{Selecting cluster } i) = \frac{mS_i}{\sum_{U_i \in U} S_i}$ m U

$\bar{n}$

S_i i

j i

S_i

$\frac{\bar{n}}{S_i}$

$\bar{n}$ $\frac{\bar{n}}{S_i}$ i

$$\pi_j(i) = \frac{mS_i}{\sum_{U_i \in U} S_i}$$

i S_i m

$$P(\text{Selecting cluster } i) = \frac{mS_i}{\sum_{U_i \in U} S_i}$$

$\bar{n}$

S_i^*

j i

$$\pi_j(j) = \frac{mS_i}{\sum_{U_i \in U} S_i} \frac{\bar{n}}{S_i^*}$$

$\frac{\bar{n}}{S_i}$

S_i^* S_i

$$Deff_{Kish} = deft^2 = deftu_s^2(1+L) = \left(1+\rho(\bar{b}-1)\right)\frac{n\sum k_j^2}{\left(\sum k_j\right)^2}$$

References

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